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DIELECTRIC PHENOMENA

Electrical Discharges in Gases

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Electrical Discharges in Gases

by S. Whitehead, B.A.

Edited
with a Preface
by E. B. Wedmore
M.I.E.E., F.Inst.P.

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ERRATA

Page 31, line 9, *for* "positive" *read* "negative."

line 10, formula *to read*

$$1=da_2dl+\int_0^l ae^{[(d\beta_1-da_2)dl+\int_0^x(\beta-\alpha)dx]}dx,$$

i.e.

$$1=\int_0^l ae^{\int_0^x(\beta-\alpha)dx}dx+d\beta_2dl_1. \quad . \quad . \quad a3.7$$

paragraph 2, formula *to read*

$$d\beta_2dl>e^{\int_0^l(\beta-\alpha)dx}da_1dl,$$

Page 71, Diagram XXII (*b*), in caption, *for* "Vites" *read* "Wires."

Page 109, formula *to read*

$$g=\frac{V\sqrt{S^2-4a^2}}{\{(a+x)(S-2a)-x^2\}\log\left(\frac{S}{2a}+\sqrt{\frac{S}{4a^2}-1}\right)}.$$

Page 110, equation *c*2.3, *to read*

$$g_v=\frac{V}{S}2S^2\frac{[S^2(f+1)+4\left(\frac{S}{2}-y\right)^2(f-1)]}{[S^2(f+1)-4\left(\frac{S}{2}-y\right)^2(f-1)]^2}. \quad . \quad c2.3$$

Page 112, last line, *for* "require" *read* "acquire."

Page 161, eleven lines from bottom, formula *to be*

$$f(x)=e^{\int_0^x(\alpha-\beta)dx}.$$

six lines from bottom, *for* $x=e$ *read* $x=l$.

Page 168, 4th line down,

for $\left(p_1-\frac{i}{2\pi u}\log\frac{r}{a}\right)$ *read* $\left(p_1+\frac{i}{2\pi u}\log\frac{r}{a}\right)$

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PREFACE

THE practice of electrical engineering is dependent upon knowledge of three classes of materials :

Insulators.
Conductors.
Magnetic materials.

Of these three the first presents a wide field for experiment and research, much of which has been carried out by engineers in an empirical fashion.

Having regard to the immense importance of the subject in this field of industry, the Electrical Research Association desires to encourage study of the fundamental phenomena, and to further this Mr Whitehead has been preparing a critical résumé of available information written from the scientific standpoint, part of which is now completed and appears in the following pages. A study of this work will show how far electrical engineers are still dependent upon vague hypothesis and disjointed data.

This volume deals with discharge phenomena in gases. Further work is in hand dealing with liquids and solids.

The present volume gives a short account of the main phenomena which have been observed in electrical discharges through gases at pressures for which the mean free path is small compared with the electrode dimensions, and a brief discussion of the theories dealing with them.

Neither the vacuum discharge nor the arc is dealt with, except incidentally, since in the former the pressures are below the range considered, while the latter is not wholly a phenomenon of gaseous ionisation.

The volume is divided into three parts. In the first, the Introduction, a very brief outline is given of the ionic theory of conduction in gases generally ; in the second Sparkover is considered ; in the third Corona, which is here used as a general term, including dark, glow, and brush discharges.

A selection of experimental results is given in the diagrams. These are of sufficient accuracy for the purposes of illustration, but for other uses the original papers should be consulted.

An Appendix is added containing mathematical and less important matter which, however, deserves mention.

Acknowledgments are due to Mr G. W. de Tunzelman for editorial assistance in preparing this work for the press.

E. B. WEDMORE,
Director and Secretary.

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Electrical Discharges in Gases

INTRODUCTION

I. CONDUCTION EFFECTS IN GASES

(*a*) Sub-discharge Fields

When a potential difference is applied between two electrodes in a gas the current passing is generally inappreciable. If, however, a current-meter of extreme sensitivity is used, or if the current is increased by using a source of constant ionising power, a current-voltage curve may be obtained, as shown in fig. 1. This curve is divisible into three parts :

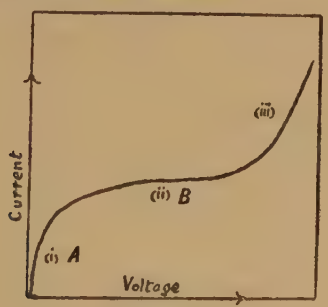


Fig. 1.

(i) Up to A the current is proportional to the voltage, *i.e.* the arrangement obeys Ohm's law.

(ii) After A the current falls off with increasing voltage until it reaches a constant " saturation " value, which continues until B is reached.

(iii) After B the current suddenly begins to increase very rapidly, attaining relatively very large values. Characteristic visual and other effects are also produced. We now say that the gas has begun to " break down " or " discharge," and fields above this point we shall call ionising fields. " Discharging fields " are fields where the discharge is fully developed.

(*b*) Ionic Theory of Conduction

The above effects are now accounted for by the presence of charged particles or ions in the gas. These will consist of

electrons and negative ions and positive ions. The latter are molecules which have parted with one or more electrons, leaving the molecules positively charged, while the negative ions are molecules which have picked up one or more electrons, leaving them negatively charged. Under the influence of the field the electrons and negative ions move towards one electrode, while the positive ions move toward the other electrode, with mean speeds determined by their own natures, that of the gas, and the strength of the field.

(i) If the field is so weak that the number of ions removed by it is small compared with the total number, the current will merely depend upon the mean speeds of the ions and on the total number per cubic centimetre, and so, other things being equal, will be proportional to the voltage. This gives part (i) of the curve in fig. 1.

(ii) As the field strength increases, the rate of increase in the current, *i.e.* of the flux of electrons and ions, decreases until the flux, and therefore the current, becomes stationary. The number of ions taken out of the gas in a given time must then be equal to the number produced in the same time. Thus the current will tend to a constant value equal to the rate at which the ions are supplied. This gives part (ii) of the curve.

(iii) When the voltage is sufficiently raised, the ions will acquire such velocities in their free paths that on colliding with neutral molecules many of the latter will split up into pairs of ions, positive and negative. These new ions will themselves produce others by collision, and so the current rapidly increases. This is part (iii) of the curve, and the effect is known as "ionisation by collision." Although this effect is, in general, the most important factor in determining the conductivity of a gas, yet secondary effects may come into play, materially affecting the phenomena in some cases. Accordingly we will next consider briefly the nature of the generation of ions in gases.

(*c*) The Generation of Ions in Gases

(A few references to standard works and important papers on the subjects of this section will be found in the Bibliography. The subjects are too vast for detailed references to be given.)

(i) *Mechanics of the Atom and Molecule*

The present view is that an atom is composed of a central nucleus positively charged, around which circulate planetary electrons of such a number that the atom as a whole is neutral. The nucleus comprises both positive charges and electrons associated with them. The mass or weight of the atom is practically determined by the positive charges (there is reason to believe that in many cases these positive charges are elementary hydrogen and helium nuclei), while the resultant nuclear charge, *i.e.* the difference between the sum of the charges on the positive core and the sum of those on the associated electrons, gives the atomic number. The atomic number is thus in a neutral atom equal to the number of planetary electrons. The number of these electrons, and therefore the atomic number, determines the chemical and physical properties of the element.

A molecule is generally considered to consist of two or more atoms which are associated by reason of their sharing one or more electrons—" valence " electrons. Precise molecular models have not, however, in the author's opinion, been very successful so far.

The electron orbits, in order to be stable, must satisfy certain conditions in addition to the ordinary classical dynamical equations of equilibrium. The first condition is that the total energy W of an orbit shall be a whole multiple of Planck's universal constant h, that is to say, $W=nh$, where n is a positive integer a1.1

(The complete set of conditions is obtained by equating the Hamiltonian phase integrals for each degree of freedom to a whole multiple of h. Usually 4 degrees of freedom are assumed giving 4 quantum numbers—the radial (determining the momentum along the radius vector), the azimuthal (determining the moment of momentum of the electron), the relativity quantum number (giving the perturbation due to the variation of mass with speed), and the magnetic (giving the moment of momentum in the direction of an external field).)

A planetary electron, in the absence of catastrophic shock, can move only in its own orbit or from one stable orbit to another, and so the energy absorbed or emitted by an atom when its structure

is modified is a whole multiple of h. We may thus consider the planetary electrons in an atom or molecule disposed at different energy levels, each characterised by a certain quantum number. According to Planck's first principle, energy may be emitted or absorbed only in discrete quanta, which we see would correspond to the difference between the initial and final energy levels of a transition. According to his second principle, energy may be absorbed continuously but only emitted in quanta; this would lead to an average energy content at absolute zero (*null-punktsenergie*), since an oscillator would always be absorbing energy and yet unable to yield it up until the quantum was reached.

It is now generally accepted that, in regions in which matter is absent, radiation is propagated in accordance with the classical laws of Maxwell. The fundamental law of quantum dynamics, that radiant energy is emitted and absorbed only in complete quanta, is no longer interpreted as meaning that the ether can carry radiant energy only in complete quanta, but that matter can deliver or absorb radiant energy only by complete quanta. It is matter, and not ether or radiant energy, which proves to be different to what we had thought. There is, accordingly, little reason for doubting that Maxwell's equations (c=speed of light),

$$\frac{1}{c}\frac{\partial X}{\partial t}=\frac{\partial \gamma}{\partial y}-\frac{\partial \beta}{\partial z}, \text{ etc.},$$

are true under the quantum theory as they were under the old dynamics. But under the old dynamics X, Y, Z had very precise meanings; they were components of a force, and an electron of charge e placed in the field would experience a mechanical force Xe, Ye, Ze. Under the quantum dynamics this cannot be the true meaning of X, Y, Z, for if it were, an electron placed in a field of radiation would be absorbing energy otherwise than by complete quanta.

Accordingly, a combination of the two views has been attempted by supposing that X, Y, and Z are related to the probabilities of a transition in the energy state of the electron; the physical force interpretation no longer being insisted upon. Bohr related quantum conceptions with statistical ideas by the correspond-

ence principle. Any oscillation of a system may be expressed as a Fourier sum of harmonic terms of ascending frequency. The amplitude coefficients of these terms are supposed to represent the probabilities of these harmonics. The quantum conditions determine the magnitude of these coefficients, *i.e.* they select certain of the statistically possible oscillations, the rest being improbable. No transition may occur which does not correspond to a term in the statistical expression.

More recently a new system of mechanics has been introduced by Schrödinger, Heisenberg, de Broglie, etc., in which energy and matter are supposed to be singularities on the wave front. On these theories the electron possesses a structure and rotation axes. The theories have already met with considerable success, the rotating electron explaining many spectral anomalies, since an orbit may now theoretically have five quantum numbers one of which is invariable.

Finally, attempts have been made to explain the quantum conditions as arising from electronic structure, and to explain the atomicity of electricity as a quantum effect in five-dimensional space.

If an atom or molecule receives (and absorbs) a quantity of energy greater than that corresponding to an electron orbit, the electron will be unable to find a stable orbit and will be lifted right out of the atom, taking with it a kinetic energy equal to the difference between the energy corresponding to its orbit and the energy supplied. The atom or molecule will be positively charged and there will be a free electron; thus two oppositely charged particles have been produced from the neutral atom. The ionised atom may subsequently pick up an electron. When this happens energy will be given out and will appear in the form of radiation. Again, subsequently, an electron so bound may fall into an inner orbit, *i.e.* to a lower energy level, again emitting radiation corresponding to the energy difference. The radiation is emitted as a quantum of energy W (determined by the initial and final energy levels) and of frequency ν, which is given by the equation $W=h\nu$ . . . . . $a1.2$

In this way an atom may emit a series of radiation quanta of definite frequencies, constituting its "spectrum."

Ionisation thus consists in the ejection of an electron by the absorption of suitable energy quanta. This energy may be obtained in mainly three ways, as described below.

(ii) *Ionisation by Impact*

When two bodies collide, for example a molecule or atom with an ion moving in an electric field, the total kinetic energy may remain the same, in which case the collision is elastic. On the other hand, the internal structure of one or both may be modified, involving a change of potential energy, and consequently of total kinetic energy. If, in the case considered, the charged ion had possessed an amount of kinetic energy greater than that corresponding to the energy level of one of the electrons of the atom or molecule, then this electron might have been ejected, ionisation taking place. It has been found that the minimum amount of energy so required to ionise an atom or molecule at a single impact can be regarded, on an average, as a constant for the gas or vapour. As the ionising agents have usually been electrons, this minimum energy quantum can be defined as Ve, where e is the electronic charge and V the accelerating potential. V is known as the "ionisation potential."

Ionisation will, however, take place at voltages (by this is actually meant energies, *i.e.* products of voltage by electronic charge) below the ionisation potential. A normal atom may collide with an ion having less than the ionising energy, but sufficient to remove an electron from one orbit to an outer one. While still in this state another collision may take place, resulting in the ejection of the electron from the already excited atom. The energy absorbed in the first collision corresponds to the energy quantum of the spectral line which would have been emitted had the transition taken place in the reverse direction. Actually such a transition would have taken place had the atom or molecule not first been ionised.

The potentials at which collisions occur, of the type involving energy transference without ionisation, as above, are known as "resonance potentials," since, as we see, they correspond to a line in the spectrum of the substance.

It appears that no ionisation occurs when the available energies

are below the lowest resonance potential; this must be so if Planck's first principle holds, but if an atom could store energy continuously ionisation might possibly be expected.

In the strong fields of a discharge ionisation by successive impact will, in most cases, be of much less importance than ionisation by single impact, owing to the smaller probability of a second collision. Each case will, however, finally be decided by the existing conditions, as many factors have to be taken into consideration.

In the case of ionisation by a given impact of an electron (or ion) with a molecule or atom, we may say that the electron (or ion) must at least have possessed the energy corresponding to a spectroscopic term for the neutral molecule, and at least this amount of energy is absorbed in the collision.

"Inelastic" and "elastic" collisions are only rough terms, since no collision is completely elastic nor is the bombarding electron (or ion) reduced to rest except very rarely. Supposing a large amount of energy available, there are still the questions as to exactly how much will be absorbed and the effects it will most probably produce. Instead of the complete removal of one electron two or more may change their energy levels, or, in the case of more complicated molecules, dissociation may occur.

The latter suggestion has recently been put forward in order to explain some anomalies in the ionisation produced by alpha particles. It has been found that in the five rare gases the ionisation increases with increasing atomic number, which would be expected from their decreasing ionisation potentials. But in the diatomic gases the ionisation is less than in any of the monatomic gases, indicating that energy is expended in other ways. The ratio of the value of the ionisation in air to that in other gases is found to vary with the velocity of the alpha particles. No such anomalies were found in the stopping powers of these gases when determined for small portions of the ranges of the alpha particles, *i.e.* for given velocities of the particles. Thus the energy primarily absorbed was normal but the ionisation which is an effect of the secondary delta rays was abnormal. Thus it appears that in this case more energy was expended in other ways than in ionisation in the case of diatomic than in the case of

monatomic gases. It is supposed that dissociation provides an extra mode of energy expenditure in the case of the former gases.

In any case, the division of absorbed energy and the proportion of energy absorbed depends upon the velocity. As has been shown in the case of alpha rays, high-speed particles lose their energy more slowly than low-speed particles. We may express the effect by saying that the effective size of atoms or molecules varies with the speed of the bombarding particles, and on the whole decreases with increase of speed for higher speeds. It has been shown in the case of slow electrons that the effective size often varies anomalously apart from increase of effective size occurring when a critical energy is reached.

The proportion of surplus energy acquired by the ejected electron will influence the field strength necessary for a given degree of ionisation. This kinetic energy varies with the velocity of the bombarding electrons and with the energy state of the target molecule or atom, but not in a simple manner. At low field strengths this matter is probably not important.

It is considered also that no collisions are altogether elastic even when the energy is less than the lowest observed resonance potential. The amount of energy so absorbed is thought to be a fairly regular function of the velocity and to increase with it, for low speeds at any rate. The amount seems always to be small compared with the energy taken up when a critical energy value is reached.

(iii) *Ionisation by Absorption of Radiation—Photo-Impact*

Radiation, according to present views, consists of a stream of discrete quanta of energy values $h\nu$, where h is Planck's constant, and ν is the frequency of the elementary spectral line considered. These quanta, on meeting an atom or molecule, may either ionise it or produce an electron transition if the quanta are of sufficient amount. The phenomenon is the same as in the collisions considered in the preceding paragraph. The quanta may be considered as particles moving with the velocity of light, c, with an energy $h\nu$ and momentum $\frac{h\nu}{c}$. In the Compton effect it is found that collisions with free electrons (which can possess only

kinetic energy when free) obey similar laws to those relating to ordinary collisions, with the exception that the light quantum cannot alter its velocity but only its energy. The phenomenon of ionisation, or excitation by radiation, is accordingly known as photo-impact.

Ionisation by a single photo-impact, or absorption of a single quantum, will occur only with radiation of sufficiently high frequency. In the case of gases the ionisation potentials vary from 4 or 5 volts up to 22 or 23 volts. Accordingly, we find that γ-rays and X-rays produce a strong ionisation in gases; Wiedemann's *Entladungstrahlen* (wave-length from 10° Å. to 1000° Å.) give a fair amount of ionisation, Schumann's rays will also ionise gases, but ordinary ultra-violet light gives a slight ionisation in a few gases only.

It is impossible for the radiation emitted by the ionised gas molecules, when recombining with electrons, to ionise the gas itself, since the spectrum it emits must, under these circumstances, correspond to lower energy potentials than the ionisation potential. Ionisation may, however, occur by the absorption of successive quanta or successive photo-impacts, just as in the case of successive electronic or ionic impacts. The former effect is of much greater importance than the latter, as has been shown by Compton, particularly at the higher pressures. Arcs have been started by illuminating a gap with appropriate radiation, although the applied voltage has been below the lowest resonance potential, cumulative ionisation of the impact type having taken place.

(iv) *Thermal Ionisation*

Strictly speaking, this is not a particular mode of ionisation, for the energy must be transmitted through the whole mass, either by radiation or by impact. It is, however, convenient to consider the phenomena separately.

We may consider ionisation as a chemical action, in which a molecule or atom is dissociated into two ions, the negative ion being an electron. There will, accordingly, be equilibrium between the undissociated particles, the positive ions, and the electrons, considered as a gas. Their partial pressures will obey the law of mass action; the energy of the reaction is the ionisation

potential, and so we can calculate the equilibrium partial pressures and, therefore, the extent of ionisation by the ordinary laws of thermo-dynamics, knowing the chemical constant.

Saha has done this, and obtained the following expression for the percentage ionisation :—

$$\log \frac{x^2}{1-x^2}P = -\frac{5050}{T}V + 2{\cdot}5 \log T - 6{\cdot}5 \quad . \quad . \quad a1.3$$

where

x=fraction of molecules ionised,
V=ionisation potential,
T=absolute temperature,
P=total pressure.

Thermal ionisation, however, does not become appreciable until very high temperatures are reached. It may be of importance in the heated region of a discharge, but otherwise would probably be negligible.

(v) *Coefficients of Ionisation by Collision*

If a gas is stressed until it begins to break down, the phenomena are ultimately due to the ionisation produced by ions moving with high velocities under the influence of the electric field. The ionisation need not necessarily be produced by direct impact; for, as we have seen, ions may be formed by successive impact, or by successive photo-impact, with the radiations emitted by neighbouring molecules or atoms. The energy for this ionisation ultimately comes from that dissipated by the ions in moving under an electric force, so we may say that the ionisation arises from the motion of ions which forms others along their paths.

The number of ions formed by an ion per centimetre of its path is known as its ionisation coefficient, and throughout this Volume α means the ionisation coefficient of the negative ions and β that of the positive ions.

Since ionisation by single impact is, generally, the most important source of ion production, the ionisation coefficients are usually considered, as a first approximation, to arise from action of this kind.

Accordingly, since the number of ions formed is proportional to the number of molecules present per cubic centimetre (since

the chance of a collision is so proportional), since the product free path × field strength determines the energy at impact, and since the probability of a given free path is inversely proportional to the pressure, we may express the ionisation coefficients as proportional to the number of molecules per cubic centimetre × a function of $\frac{\text{field strength}}{\text{pressure}}$, or if

δ = density of the gas under consideration,

X = field strength,

$$\alpha = \delta f\left(\frac{X}{\delta}\right), \quad \beta = \delta g\left(\frac{X}{\delta}\right) \quad . \quad . \quad . \quad a2.1$$

where f and g are arbitrary functions for a given gas.

We may also put

$$\alpha = pf\left(\frac{X}{p}\right) \quad \text{and} \quad \beta = pg\left(\frac{X}{p}\right) \quad . \quad . \quad a2.2$$

where p = pressure.

Unfortunately, the forms of these functions are not yet known, but all discharge phenomena must depend upon them.

The ideal would be to connect α and β with the critical potentials and molecular constants of the gas, but very little progress has been made in this direction.

All we can say at present is that, for lower velocities of the ions $\left(i.e.\text{ smaller values of } \frac{X}{p}\right)$, the electron, having a smaller mass, is a more efficient ionising agent, but for high values of $\frac{X}{p}$ the heavier ion (the positive usually) is better; while for α rays and β rays the former, which are positive ions, or protons, are incomparably more efficient than the latter, which are electrons.

(vi) *Breakdown Strength*

Suppose an ion is moving in a uniform field of strength X and that the mean free path of the ion is l.

If Xle = energy necessary for ionisation, a large majority of collisions should result in ionisation. We might very well call X the breakdown strength of the gas. If V is the ionisation potential,

$$X = \frac{V}{l}.$$

This simple definition leads to difficulties. In the first place, the mean free path of the ion depends upon X. In the second place, owing to the distribution of velocities, ions with sufficient energies would be found at lower field strengths than that calculated as above. In the third place, so long as there were ions with energies greater than the resonance potential, ionisation by successive direct impact and photo-impact might occur. The definition has, however, been given.

Nevertheless, in practice no such precise definition is generally employed. In most cases the term is taken to mean that strength which in a large number of cases appears to determine the limit of ionisation by collision. As it is an empirical term it will be dealt with in the section on corona, where it is most frequently employed.

(vii) *Radiations emitted by the Discharge*

As we have seen, an ionised atom, or molecule, tends to take up an electron, and in so doing emits a spectral line or series of spectral lines. A discharge will therefore emit a considerable amount of radiation, which, we have also seen, may produce further ionisation.

A spectroscopic examination of a discharge thus gives information as to the nature of the ionisation taking place. From the limit of the spectroscopic "terms," which correspond to the energy levels of the atom, we may calculate the ionisation potential corresponding to the energy level, giving rise to the spectral series. Spectral series, resonance, and ionisation potentials are of course different aspects of the same phenomenon, namely, electron transitions.

We may note here that in the spark discharge we find the spectrum of the ionised atom emitted, indicating a large amount of double ionisation in the discharge.

The radiations emitted by discharges cover all the visible spectrum and extend beyond the ultra-violet. Wiedemann's *Entladungstrahlen* are given out by the spark and silent discharges.

II. TYPES OF DISCHARGES

(*a*) Discharge a Form of Instability

In Section I (*a*) it was stated that when the current began to rise rapidly after reaching the saturation value (*i.e.* when ionisation by collision began) " discharging " began. One may speak of its beginning then, although the full discharge may not be developed. As the voltage is raised we reach a point where there is a sudden jump in the current, and characteristic effects appear, such as glows, brushes, sparks, and their accompanying phenomena. This instability marks the formation of a discharge.

Initially, ionisation by collision is due to one kind of ion only, generally the negative, but when the voltage is raised both sets of ions produce others in large quantities and, after a further rise in voltage, such numbers of ions are formed that new factors, due to space charges, etc., become important, and the current suddenly jumps to a new position of equilibrium.

(*b*) Characteristics of the Types

The characteristics of these new equilibria fall into five classes : the dark discharge, the glow, the brush, the spark, and the arc. Which one appears depends upon the conditions of the experi-

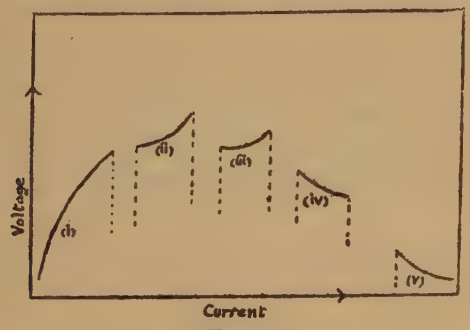


Fig. 2.

ment ; one may pass into the other with further rise of voltage, or sometimes automatically, but if more than one appears their order follows that given. Their characteristic voltage/current curves are shown in fig. 2.

(i) In the dark discharge the current, compared with other discharges, is small, and increases relatively slowly. No visual effects are observed, but chemical action takes place.

(ii) The glow is the most frequent initial discharge in non-uniform fields. It is not found in uniform fields. A glow appears in the intense parts of the field, accompanied by a hissing noise. It is also called "corona." The current, which is larger than for (i), increases fairly rapidly with voltage and tends quickly to give brushes and finally a spark.

(iii) The "brush" discharge is midway between a glow and a spark. It is practically always accompanied by a glow, and is a development of it. It consists of a number of small sparks ending in air, and starts from irregularities or abnormalities in the conductors. It is the first form of instability of the glow, and for practical purposes can be treated as an abnormality of the latter.

(iv) The "spark" is the commonest discharge, as it may always occur, except at very low pressure, if there is sufficient voltage. It is unstable, *i.e.* the passage of the current lowers the voltage. Actually it is a sudden rush of electricity through the gas, and the amount of the current is, in general, limited only by the capacity of the testing machine. The spark is the most complete form of "breakdown."

(v) The arc is mentioned here only for completeness. It is not a form of ionisation by collision, but arises from other discharges, when the electrodes are sufficiently hot to supply electrons for the current, and to vapourise the electrodes which supply positive particles, which heat the cathode by impact. The current density is enormously high for the voltage compared with other discharges, and is unstable unless limited by an external resistance.

III. GENERAL PRINCIPLES OF DISCHARGES

(*a*) Nature

From the foregoing it is seen that discharges have the following properties :—

(i) They are due to ionisation by collisions of positive and negative ions.

(ii) They are a form of instability due to the fact that, when sufficient ions are formed, the current suddenly adjusts itself to suit new conditions.

(iii) Their voltage/current relations must depend on the coefficients α and β. Since these are different for non-uniform fields, positive and negative discharges will, on the whole, be different.

Further, since α and β depend on the density (*i.e.* pressure, in most cases), discharges will depend both on the character of the field (as determined by the shape of the electrodes) and on the density of the gas.

(iv) Discharges will also depend on the nature of the gas.

(*b*) General Discharge Theorem

(i) *Theorem*

As the relations of the coefficients α and β of ionisation by collision are little known, and as, consequently, the calculation of discharge fields is very difficult, there is no precise theory of discharges. One or two relations and a few rather vague principles can, however, be given, although the mass of the information is largely empirical.

The most general and important relation is that connecting discharge potential and density for similar systems.

Geometrically similar systems will discharge at the same potential if the products of their leading dimension and mass of gas per cubic centimetre are the same; or

The discharge potential is a function of the product of density and dimension for similar systems; [1] or in a formula,

$$V=f(\delta a) \quad . \quad . \quad . \quad . \quad a3.1$$

where

V=discharge potential,
δ=density (=mass of gas per cubic centimetre),
a=dimension.

(ii) *Proof*

Let σ =surface density of electricity over the electrodes A, B.
σ'=surface density of electricity over the electrodes A′, B′.

[1] Townsend, *Electrician*, 71, 348 (June 1913).

Let ds, ds' be elements of surface area for A, B, and A′, B′ respectively.

r, r' be the distances of specified points from ds and ds'.

the two systems be similar with their dimensions in the ratio $1 : k$.

v (the potential at any point in system A, B)$=\int\frac{\sigma ds}{r}$.

v' (the potential at any point in system A′, B′)$=\int\frac{\sigma' ds'}{r'}$

$=\int\frac{k\sigma' ds}{r}$.

For $r'=kr$, $ds'=k^2ds$.

If $\sigma=k\sigma'$, the electrodes will have the same potentials in each case, and the distribution of potential will be similar.

The number of ions generated by a negative ion in system A, B is αds, where ds is an element of the ionic path, *i.e.* is

$\delta f\left(\frac{x}{\delta}\right)ds$, where δ=density, x=field strength, *i.e.* is $\delta f\left(\frac{1}{\delta}\frac{\partial v}{\partial s}\right)ds$.

Similarly for A′, B′ the number of ions generated by a negative ion in ds' is

$$\delta' f\left(\frac{1}{\delta'}\frac{\partial v'}{\partial s'}\right)ds' \text{ (where } \delta'=\text{density)}=k\delta' ds f\left(\frac{1}{\delta' k}\frac{\partial v}{\partial s}\right),$$

for $v=v'$, as above, and $s'=ks$.

If $\delta=k\delta'$, the expressions for both systems are identical. Thus, if sufficient ions are generated by the given potential difference in the case of A, B, the same number per path element will be generated for the same potential difference in the case of A′, B′ if the densities are inversely as the dimensions. Since the discharge depends on the requisite number of ions being formed, the theorem is proved. *In the term density is included the effect both of pressure and temperature.*

(iii) *Theorem for Gradient*

If we can express a characteristic gradient in the system in terms of the voltage and a linear dimension (*e.g.* in the wire and

cylinder the gradient at the wire surface will be $\frac{V}{\lambda a}$ where

V=voltage,
λ=log (ratio of radii of wire and cylinder),
a=radius of wire),

then we can transform the general discharge theorem above into the form

$$\frac{\text{Gradient}}{\text{Density}} = \text{function (dimension} \times \text{density)} \quad . \quad a3.2$$

For the gradient $= \frac{\text{voltage}}{\text{dimension}} \times$ quantity independent of dimension, and dividing equation $a3.1$ throughout by (dimension $\times$ density, the theorem $a3.2$ is obtained.

(c) Townsend's Continuity Theorem *

We can calculate the conditions necessary for the continuance of a discharge, neglecting recombination and other losses of ions. Let $n_1 n_2$, uv, $\alpha_1 \beta_1$ be the number of negative and positive ions per cubic centimetre, their velocities and ionisation coefficients respectively. Then, for the maintenance of the discharge, n_1 and n_2 must not decrease with lapse of time. Therefore

$$\frac{\partial n_1}{\partial t} = -\frac{\partial}{\partial x}(n_1 u) + \alpha n_1 u + \beta n_2 v = 0,$$

$$\frac{\partial n_2}{\partial t} = \frac{\partial}{\partial x}(n_2 v) + \alpha n_1 u + \beta n_2 v = 0,$$

where

t=time

and

$$n_1 u + n_2 v = c,$$

where ce=the current i and x refers to the discharge path.

On solving these equations (see Appendix I) and inserting the conditions at the electrodes, viz.,

$\left.\begin{matrix} x=0 \\ n_1=0 \end{matrix}\right\}$ at negative electrode, $\left.\begin{matrix} x=l \\ n_2=0 \end{matrix}\right\}$ at positive electrode,

we have

$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx = 1 \quad . \quad . \quad . \quad a3.3$$

* Townsend, *Electricity in Gases*, p. 428.

This condition must be satisfied in order that the discharge may continue. It is also the limiting condition as far as the production of ions is concerned. If they are produced any slower the discharge must die away. It is therefore to be expected that this condition gives the *critical field strength* and minimum voltage at which the discharge may be maintained, *i.e.* the "discharge voltage." When considering sparkover we shall find that the equation *a*3.3 agrees in a particular case with the condition for discharge calculated from a more direct point of view.

Over the part of the discharge path where no ionisation is occurring the integral in *a*3.3 is zero. Thus in the case of a limited discharge, such as corona, the integral need only be taken from the corona-forming electrode to the point at which the discharge dies away, *i.e.* at which ionisation ceases. In the case of the spark the integral must be taken over the whole or the greater part of the distance between the electrodes, since a long path of ionisation is characteristic of the spark.

If we assume that the ratio $\frac{\beta}{\alpha}=k$, a quantity independent of the field strength *a*3.3 reduces to

$$\int_0^l \alpha dx=\frac{\log k}{k-1}=\text{const.} \quad . \quad . \quad . \quad a3.4$$

This equation has been given by Schumann * as the general discharge condition. It means that a discharge will occur when the total number of ions formed per ion over the whole discharge path reaches a given value.

(*d*) Asymmetry of Positive and Negative Discharges in non-Uniform Fields

The continuity theorem, as above, has been shown to give the equation

$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx=1 \text{ (see } a3.3) \quad . \quad . \quad . \quad a3.5$$

as a necessary condition for discharge, where α and β are the ionising coefficients of the positive and negative ions, and l is the

* W. O. Schumann, *Zeit. für Techn. Phys.* (1925).

length of the discharge path. Suppose now that the field strength is increased near the positive electrode for a distance of dl, then in this region α and β will be increased to $\alpha+d\alpha_1$, $\beta+d\beta_1$ (say). The condition then becomes

$$1=\int_0^l \alpha e^{\int_0^x(\beta-\alpha)dx}dx+e^{\int_0^l(\beta-\alpha)dx}d\alpha_1 dl \quad . \quad . \quad a3.6$$

The quantity $d\beta_1$ does not occur, therefore there is no advantage in an increase in ionising efficiency of the positive ions.

Suppose now α and β are increased to $\alpha+d\alpha_2$, $\beta+d\beta_2$, for a distance dl near the positive electrode, the condition becomes

$$1=d\alpha_2 dl+\int_0 \alpha^{(d\beta_2-d\alpha_2)dl+\int_0^x(\beta-\alpha)dx}dx,$$

i.e.

$$1=\int_0^l \alpha^{\int_0^x(\beta-\alpha)dx}dx+d\beta_2 dl_1 \, . \quad . \quad . \quad a3.7$$

i.e. the increased efficiency of the negative ions does not matter in this case.

From this calculation we can draw two inferences :

1. From the form of the equation

$$1=\int_0^l \alpha e^{\int_0^x(\beta-\alpha)dx}dx$$

we see that the condition is not symmetrical in α and β, therefore *negative and positive discharge will, on the whole, occur at different voltages.* For uniform fields we have seen that the condition reduces to a symmetrical one, so that this difference between positive and negative discharges is a mark of non-uniform fields.

2. If

$$d\beta_2 dl>e^{\int_0^l(\beta+\alpha)dx}d\alpha_1 dl,$$

i.e. if

$$\frac{d\beta_2}{d\alpha_1}>e^{\int_0^l(\beta-\alpha)dx},$$

or approximately, if

$$\frac{1}{\beta}\frac{d\beta}{dX}>\frac{1}{\alpha}\frac{d\alpha}{dX}\,* \quad . \quad . \quad . \quad a3.8$$

* Townsend, *Electricity in Gases*, p. 447.

where X is the field strength, then an increase of voltage near the negative electrode will have more effect than an increase near the positive electrode, or, in other words, the negative discharge will occur at a lower potential if

$$\frac{1}{\beta}\frac{d\beta}{dX} > \frac{1}{\alpha}\frac{d\alpha}{dX},$$

and *vice versa*.

But α and β depend on the pressure also, so that we should expect that at some pressures the negative discharge would take place earlier, and that at other pressures the positive discharge would take place earlier for the same arrangement: and in practice this is found to be the case.

Taking a few rough experimental values for α and β, Townsend comes to the conclusion that when $\frac{X}{p} > 150$ (X=field strength in volts/cm. and p=pressure in mm.) $\frac{1}{\beta}\frac{d\beta}{dX} > \frac{1}{\alpha}\frac{d\alpha}{dX}$. Thus for fields where the forces are of this order the negative discharges will be obtained at lower potentials than the positive.

SPARKING

I. GENERAL PRINCIPLES

(*a*) Path of Ionisation

The appearance of the spark is well known. When the voltage is sufficiently raised there is a sudden flash, accompanied by a crackling sound, and the condenser system of the conductors is discharged (at least momentarily so). The visual appearance shows that very intense ionisation occurs along a path which extends over the whole, or a large part, of the distance between the electrodes. This long path of ionisation is characteristic of the spark.

(*b*) Fall of Potential

Directly the sparking has begun there is an immediate drop of potential, the fall being higher the larger the charge taken by the spark. This is also characteristic of the spark, and gives it its instability. In other discharges ionisation takes place throughout a certain volume, and a higher voltage is necessary to increase this volume (and therefore the current), so that a certain measure of stability is obtained. When, however, the ionisation volume exceeds a certain size, the space charge begins to favour the ionisation and a spark suddenly shoots across.

The conditions for sparking are, that sufficient ions should be formed, and that they should be sufficiently favourably placed.

(*c*) Continuity Theorem

In the Introduction the following condition was obtained for the maintenance of a discharge :—

$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx = 1 \quad . \quad . \quad . \quad b1.0$$

where α and β have their usual meanings and the integration is taken over the discharge path l.

Since this is a condition of maintenance of a discharge it does not directly refer to the spark, which is an unstable discharge. We will suppose that a stable discharge, *e.g.* corona, is formed. The discharge is limited in volume, so that the above condition holds. As the voltage is increased the volume of the discharge

increases, but, owing to the modification of the field by the space charge, *b*1.1 still holds. Finally, however, when the stable discharge has sufficiently increased, the condition *b*1.1 cannot be fulfilled except with an infinite current. A spark now occurs.

Thus, although *b*1.1 does not directly refer to the spark, yet it must hold just before a spark takes place, and so the condition applies when the modification of the field by the space charges is taken into account.

Schumann, as we have seen, gives the simpler form:

$$\int_0^t a\,dx = \frac{\log k}{k-1} \quad . \quad . \quad . \quad . \quad b1.01$$

where β is assumed $= ka$.

This integration represents the points of view mentioned above, since it expresses the fact that a given number of ions per ion must be formed along the discharge path for the discharge to path. The distribution is taken into account by the further condition, given above, that for the spark the field must be such that a small increase of voltage will make *b*1.0 impossible.

The condition *b*1.0 is a condition for the maintenance of the discharge. The voltage for sparkover calculated from it will therefore correspond, not to the voltage at which a spark starts, but to the voltage after the voltage drop mentioned above has taken place.

In uniform fields corona does not occur, so that condition *b*1.0 should give the sparkover condition. As we shall see later, we find that *b*1.0 also gives the condition that the current should be infinite. This should occur in all cases where the spark only occurs; *i.e.* in such cases the theoretical corona current should be infinite, but, probably owing to difficulties of calculation, the view has not been worked out.

(*d*) Low Pressure Limitation

It was mentioned in paragraph (*b*) that, for a spark to pass, there must be sufficient favourably placed ions. In the light of this we can understand why the spark, as we know it, does not take place at very low pressures. This is due to the fact that insufficient ions may be formed, owing to the long free paths of

molecules and ions. Ionisation occurs in jumps, while the spark requires a path of virtually continuous ionisation, for which small free paths, such as occur with higher pressures, are necessary.

II. UNIFORM FIELDS

(*a*) Definition

By a uniform field in this connection is meant any field in which the strongest parts of the field are uniform over a fair proportion of the electrode area. Parallel plates are good examples, also spheres at a spacing not more than one-half the radius of the smallest. As regards this latter arrangement, Baille,[3] Paschen,[4] and Peek [5] have found that, below this spacing, increase in the size of the spheres has no effect on the sparking potential.

Since the spark in general begins in the strongest part of the field, and since it is of small cross-section, it is clear that, provided the strongest parts are uniform, the extent of the uniformity of the field need not be very great.

(*b*) Theoretical Considerations for Ordinary and Low Pressures

By " ordinary and low " pressures here is meant pressures from a few millimetres up to a few atmospheres.

(i) *Paschen's Law*

1. deduction from general theorem

The general theorem of discharges given in the Introduction, III (*b*), can be immediately applied to uniform fields. Here the only dimension concerned is the length of the field, for the cross-section obviously does not count. But the length of the field is the spark length, so we can say that *the spark potential is a function of the product of spark length and density*, i.e.

$$V=f(\delta l) \quad . \quad . \quad . \quad . \quad b1.1$$

[3] Baille, *Ann. de Chim. et de Phys.*, (5), **25** (1882).

[4] Paschen, *Wied. Ann.*, **37** (1889).

[5] Peek, *Dielectric Phenomena* (see Tables XXI–XXIV, and compare values for spacings =3 mm.).

2. DEDUCTION FROM CONTINUITY THEOREM

Suppose now we take Townsend's continuity expression,

$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx = 1 \quad . \quad . \quad . \quad b1.0$$

Since we have a uniform field, α and β are independent of x.

$$\therefore \int_0^l \alpha e^{(\beta-\alpha)x} dx = 1,$$

$$\therefore \left[\frac{\alpha}{\beta-\alpha} e^{(\beta-\alpha)x}\right]_0^l = 1,$$

$$\therefore \frac{\alpha\left(e^{(\beta-\alpha)l}-1\right)}{\beta-\alpha} = 1,$$

i.e.

$$\alpha = \beta e^{(\alpha-\beta)l} \quad . \quad . \quad . \quad . \quad b1.2$$

where α and β are the coefficients of ionisation by collision, and l=spark length.

But $\alpha = \delta f\left(\frac{X}{\delta}\right)$ $\beta = \delta g\left(\frac{X}{\delta}\right)$ (see formula $a2$), where X=field strength and δ=density.

For a uniform field, $Xl = V$=the potential difference. Therefore substituting,

$$\log \frac{f\left(\frac{V}{\delta l}\right)}{g\left(\frac{V}{\delta l}\right)} = \delta l\left[f\left(\frac{V}{\delta l}\right) - g\left(\frac{V}{\delta l}\right)\right].$$

Thus V is a function of (spark length $\times$ density), as in $b1$.

3. DEDUCTION FROM CURRENT FUNCTION

Lastly, we can obtain the same law from another and very interesting point of view. If we consider the current between two parallel plates, l cm. apart, subjected to radiation, so that there is a photo-electric current i_0 before ionisation by collision begins, we get for i the current for a P.D. of V_1,

$$\frac{i}{i_0} = \frac{(\alpha-\beta)e^{(\alpha-\beta)l}}{\alpha-\beta e^{(\alpha-\beta)l}}^{6} \quad . \quad . \quad . \quad b1.3$$

(See Appendix II)

[6] Townsend, *Electrician* (3rd April 1903); *Electricity in Gases*, p. 314.

The continuity condition above mentioned becomes $\alpha=\beta e^{(\alpha-\beta)l}$ for uniform fields. If we substitute this value we have $i=\infty$. This result is immediately explicable if we have sparking here, for, as has been stated, sparking is an instability; and such an instability is well represented mathematically by the current becoming infinite.

This law, viz. that the sparking potential is a function of the product of the density of the gas and the spark length, is known as Paschen's law,* and is one of the few laws of sparking which holds over a considerable range.

It can also be expressed thus: The sparking potential is a function of the mass of gas between the electrodes . . *b*1.4

For the mass of gas is proportional to the product of density and electrode separation, since we may take any particular cross-section. In comparing different gases, the mass of gas must be replaced by the actual number of molecules.

It is very interesting to note how this law can be deduced from three such formally different methods as from the general discharge theorem, the continuity theorem, and the current-function for parallel plates.

(ii) *Guye's Note on the Current-function for Plates*

Guye [7] suggests that, from the formula for currents between parallel plates, viz.

$$\frac{i}{i_0}=\frac{(\alpha-\beta)e^{(\alpha-\beta)l}}{\alpha-\beta e^{(\alpha-\beta)l}} \text{ (as in } b1.3),$$

three discharge potentials can be deduced, since i becomes infinite when

$$\left.\begin{array}{ll}(1) & \alpha=\beta e^{(\alpha-\beta)l}\\ (2) & \beta=\alpha e^{(\beta-\alpha)l}\\ (3) & \alpha=\beta=\dfrac{1}{l}\end{array}\right\} \quad . \quad . \quad . \quad b1.5$$

(1) and (2) seem merely to express the fact that there might be a difference between positive and negative discharges. But in this

* It is often given as sparking potential=function of (pressure×spark length), but the above is a more general form.

[7] Guye, *Arch. des Sciences*, 42 (1916).

case there is no such difference, *e.g.* A.C. makes no difference. (3) gives two conditions which must be simultaneously satisfied, and probably it never occurs.

(iii) *Deduction of Formula for Spark Potentials* [8]

(See also Appendix III)

From the relation $\alpha=\beta e^{(\alpha-\beta)l}$ we should, if we knew α and β in terms of pressure and field, be able to deduce a formula for the spark potential. This in the present state of knowledge cannot be done at all satisfactorily. An approximate attempt can be made as follows :—

Let c be the number of ionic paths greater than x. Then the number of paths between x and $x+dx$ is dc. If θdx is the probability of a collision during dx,

$$\frac{dc}{c}=\theta dx.$$

$$\therefore \quad c=Ce^{-\theta x},$$

where C is a constant of integration.

The total number of paths is given by $x=0$; therefore the total length of these paths is $C\lambda$, where λ is the mean free path.

But the total length

$$=\int_0^\infty x dc=\frac{C}{\theta}.$$

$$\therefore \quad c=Ce^{-\frac{x}{\lambda}}.$$

If the potential drop over the path x is the ionisation potential, all paths of length greater than x will end in ionisation. This value of x is given by the equation

$$x=\frac{V_0}{X},$$

where X is the field strength and V_0 is the ionisation potential. Therefore the number of ionising collisions per centimetre is a, and is equal to

$$Ape^{-\frac{V_0}{\lambda X}},$$

[8] Townsend, *Electricity in Gases*, p. 292.

where A is a constant. Since there is no difference between positive and negative discharges in uniform fields, we may put $\frac{\alpha}{\beta}=k$, which gives

$$\therefore \quad l=\frac{1}{\alpha}\frac{k}{k-1}\log k \text{ (from } b1.2 \text{ by substitution).}$$

Let p be the pressure, then

$$pl=\frac{1}{A}e^{\frac{Bpl}{V}}\frac{k}{k-1}\log k \quad . \quad . \quad . \quad b1.6$$

where B is a constant, since λ varies as $\frac{1}{p}$ where V is the sparking potential and l is the spark length. This formula gives V in terms of pl, but it is adequate only over very small ranges. The formula for α does, however, hold for weak fields, but there is no evidence that it does so for discharge fields.

If we take the formula

$$\alpha l=\frac{k\log k}{k-1},$$

and make the assumption that $\alpha=\lambda(X-X_0)$, where λ is a constant, and X_0 = constant = breakdown strength at given pressure, so that

$$X=X_0+\frac{k}{\lambda l(k-1)}\log k, \quad . \quad . \quad b1.61$$

i.e.

$$X=X_0+\frac{a}{l} \quad . \quad . \quad . \quad . \quad b1.62$$

or

$$V=a+X_0 l \quad . \quad . \quad . \quad . \quad b1.63$$

This equation ($b1.63$) agrees with experimental results over quite wide ranges (see (c) (i)), and in the form $b1.62$ has been applied to corona in Townsend's theory with success. On the other hand, it does not tell the whole story and is almost entirely empirical.

Schumann [47] (see also [40]) has suggested the following expressions for α, viz.,

$$\alpha=Ae^{-\frac{B}{X}}, \quad Ae^{-\frac{B}{X^2}}, \quad \frac{A}{X}e^{-\frac{B}{X^2}}, \quad \frac{A}{X^2}e^{-\frac{B}{X^2}}.$$

[47] Schumann, *Archiv für Electrotechnik*, **12** (1923).

They give quite good agreement over limited ranges, but have no wider an application than *b*1.63. As far as can be gathered they are empirical except the first, which, of course, is identical with the formula

$$\alpha = Ape^{-\frac{V_0}{X}},$$

as calculated above.

(*c*) Experimental Results for Ordinary and Low Pressures

(i) *For the Range where Boyle's and Charles' Laws hold*

Where Boyle's and Charles' laws hold it is immaterial whether we express Paschen's law as :

$$\left.\begin{array}{l}\text{Sparking potential} = \text{function (pressure} \times \text{spark length)} \\ \text{or Sparking potential} = \text{function (mass of gas, } i.e. \text{ number of molecules between electrodes)}\end{array}\right\} \quad b2.1$$

Further, if Paschen's law holds, and if we know the form of the function, we shall have the same relation between sparking potential and spark length for a given pressure ; the dependence on both length and pressure being comprised in the one relation.

AIR

The following experimenters have verified Paschen's law for air, and investigated the form of the dependence of spark potential on $p \times l$ (p=pressure in mm., l=spark length in cm.) for values of $p \times l$ from 0·1 to 20,000: Paschen,[9] Baille,[10] Kelvin,[11] de la Rue and Müller,[12] Carr,[13] Townsend and Hurst,[14] Guye and C. Stancescu,[15] Peek,[16] Müller,[17] Spath,[18] Liebig,[19] Guye and Mercier,[20] Schumann,[26]

[9] Paschen, *Wied. Ann.*, **37** (1889).
[10] Baille, *Ann. de Chim. et de Phys.*, **5**, 25 (1882).
[11] Kelvin, *P.R.S.*, **10** (1860).
[12] de la Rue and Müller, *Phil. Trans.*, **171** (1880).
[13] Carr, *Phil. Trans.*, **201** (1903).
[14] Townsend and Hurst (see *Electricity in Gases*).
[15] Guye and C. Stancescu, *Arch. des Sciences*, **43** (1917).
[16] Peek, *Trans. A.I.E.E.* (1913).
[17] Müller, *Ann. de Phys.*, **28** (1909).
[18] Spath, Schumann, "Durchbruchfeldstarke von Gasen," p. 173.
[19] Liebig, *Phil. Mag.*, **24** (1887).
[20] Guye and Mercier, *Arch. des Sciences* (1920).
[26] Schumann, *Archiv für Electrotechnik*, **11** (1922).

Rudy,[21] Quincke,[23] Czermak,[24] Villard,[25] Wolf.[22] In Diagrams I–VI some of their results have been plotted for comparison in the form sparking potential *versus* pressure×spark length. As the range is too large to put them in one diagram, the logarithms have been plotted for a mean curve in Diagram I, the frames showing the ranges of the succeeding diagrams, where actual values have been given on scales convenient for the ranges concerned.

For Diagrams II and III (for higher pressures) the agreement, considering the number of years over which the experiments are spread, is remarkable, and, since most of the values have been reduced from experiments at different constant pressures, Paschen's law is fully confirmed. For any particular range the results lie practically on a straight line, but the whole curve of results for Diagrams II and III shows a slight curvature towards the abscissæ. For lower values of the product $p \times l$ (pressure ×spark length) divergences between different experimenters occur which, as the product decreases, become very considerable (see Diagrams IV and V). At the same time the sparking potentials fall off more rapidly, reaching a minimum (according to Carr [13]) for values round about half a millimetre. The curve then rises rapidly for a further reduction of the product. Although the agreement between different observers is small for low values of the product *pl*, yet they all agree that Paschen's law still holds to a fair degree of accuracy. Near the minimum of sparking potential this is no longer true.

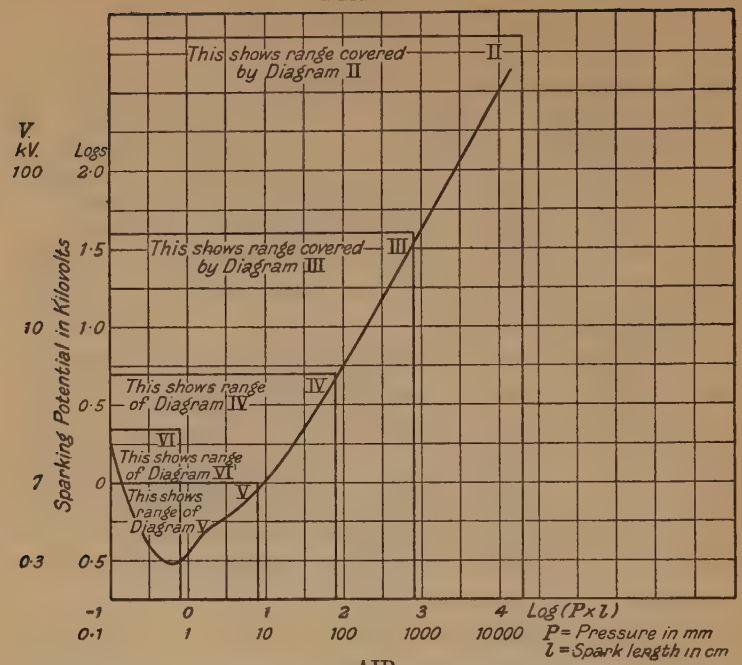
Thus, for the ranges of values of *pl* considered in Diagrams II–V, Paschen's law can be taken as proved. However, no general formula has been devised to represent the whole curve, but for a limited range a straight line can be found to represent the results with sufficient accuracy. Accordingly, in dealing with any particular problem, we can usually assume that

$$V = A + Bpl \quad . \quad . \quad . \quad . \quad . \quad b2.2$$

---

[21] Rudy, *Rev. Gen. de l'Electricité* (1923).
[23] Quincke, *Wied. Ann.*, **19**, 545 (1883).
[24] Czermak, *Wien. Ber.*, **97** (IIa), 507 (1888).
[25] Villard and Abraham, *Compt. Rend.*, **153** (1911).
[22] Wolf, *Wied. Ann.*, **37**, 306 (1889).

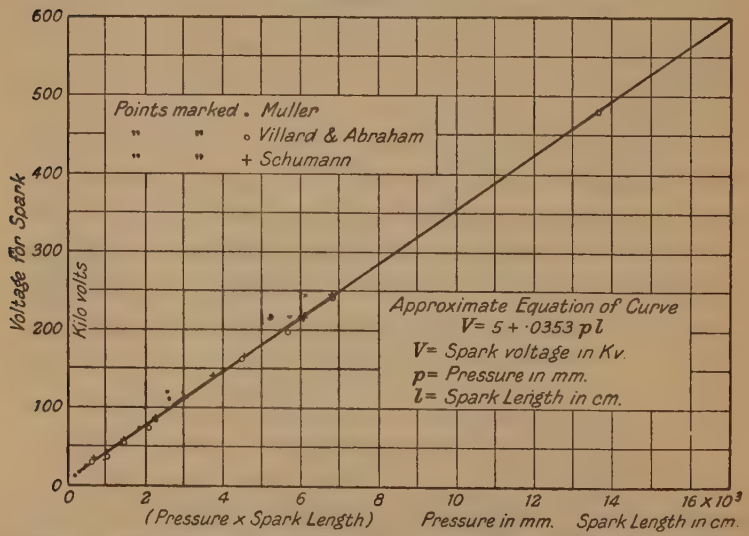
DIAGRAM I



AIR

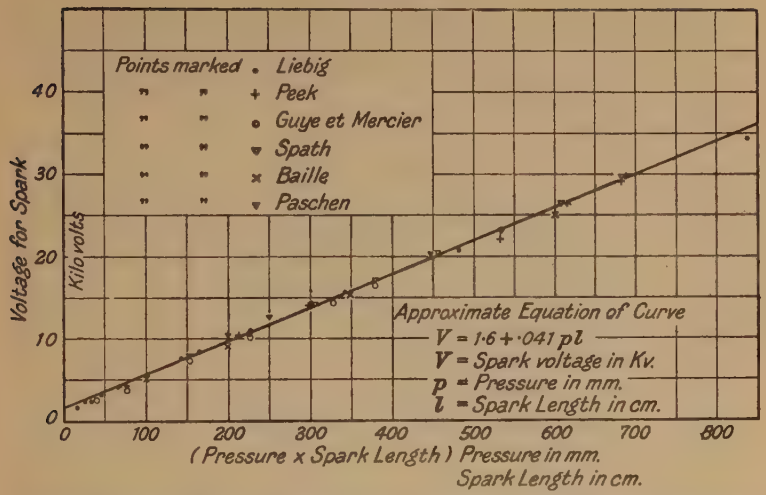
Relation between Log (Sparking Potential) and Log (Pressure × Spark Length) for uniform Fields. Logs are used to obtain the Readings in one Diagram. The Curve is a mean. The Frames marked II, III, IV, V, VI show the Ranges covered by the succeeding Diagrams II–VI, where the actual Readings are plotted on scales convenient for the Range dealt with.

DIAGRAM II



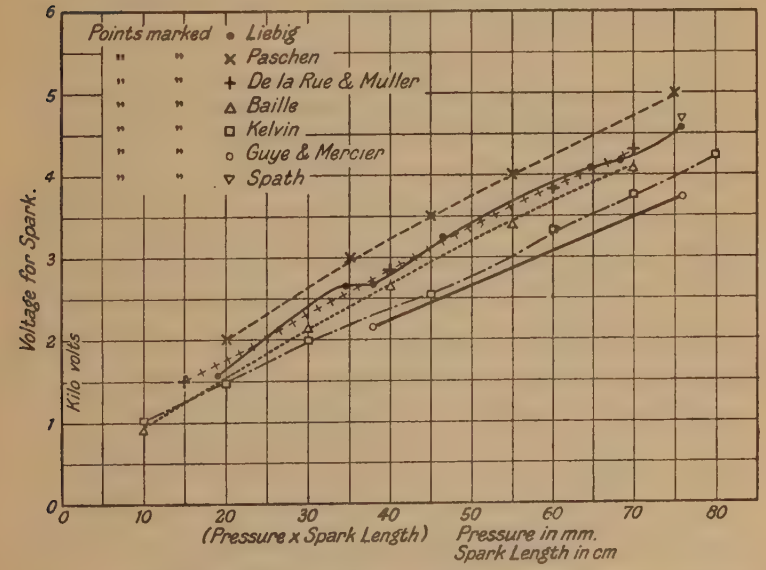
Relation between Spark Potential and Product (Spark Length × Pressure) for uniform Fields in Air. Corresponding to Range represented by Frame marked II in Diagram I.

DIAGRAM III



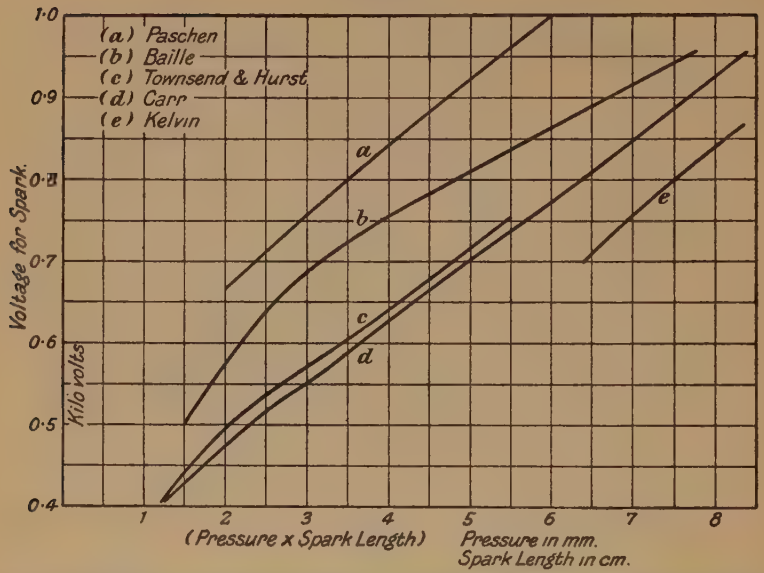
Relation between Spark Potential and Product (Spark Length × Pressure) for uniform Fields in Air. Corresponding to Range represented by Frame marked III in Diagram I.

DIAGRAM IV



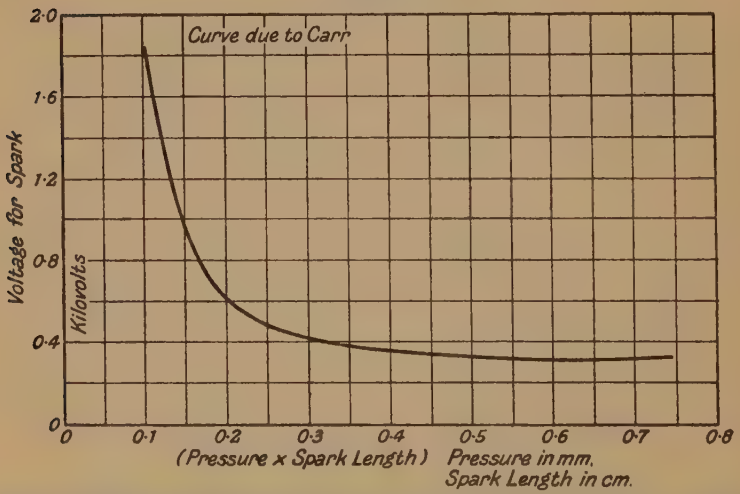
Relation between Spark Potential and Product (Spark Length × Pressure) for uniform Fields in Air. Corresponding to Range represented by Frame marked IV in Diagram I.

DIAGRAM V



Relation between Spark Potential and Product (Spark Length × Pressure) for uniform Fields in Air. Corresponding to Range represented by Frame marked V in Diagram I.

DIAGRAM VI



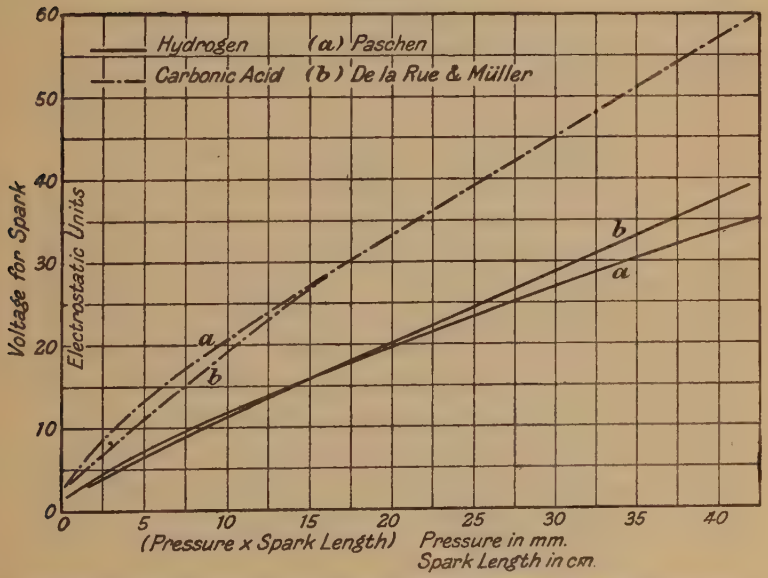
Relation between Spark Potential and Product (Pressure × Spark Length) for uniform Fields in Air. Corresponding to Range represented by Frame marked VI in Diagram I.

where V=sparking potential, p=pressure, l=spark length, A and B are disposable constants. In Diagrams II and III are given appropriate formulæ of the above form, representing the points on the particular diagram.

OTHER GASES

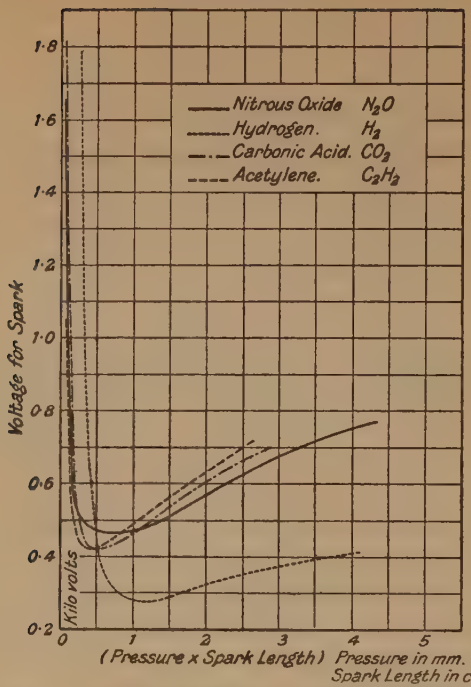
Diagrams VII–IX show curves obtained in the same way for hydrogen and carbonic acid by Paschen,[9] and de la Rue and Müller [12] at higher pressures, and for nitrous oxide, hydrogen, carbonic acid, acetylene, hydrogen sulphide, sulphur dioxide, and oxygen by Carr [13] for a lower range. On Diagram IX is shown a curve for air for comparison. The results are very similar in general tendencies, and what has been said about their general form in air applies to other gases. On the whole, those with higher molecular weights give higher sparking potentials, but there are exceptions, *e.g.* carbonic acid is lower than

DIAGRAM VII



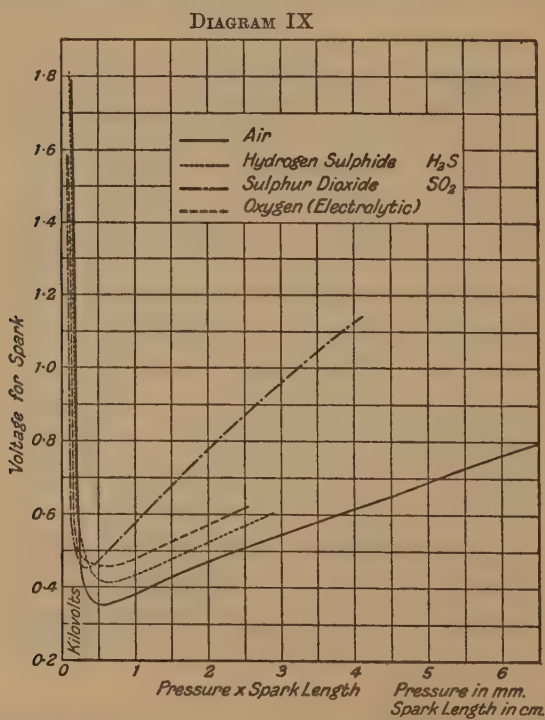
Relation between Spark Potential and Product (Pressure × Spark Length) for uniform Fields in Hydrogen and Carbonic Acid at Higher Pressure.

DIAGRAM VIII



Relation between Spark Potential and Product (Pressure × Spark Length) for uniform Fields in N_2O, H_2, CO_2, C_2H_2 at Low Pressure (Curve due to Carr). A Constant Spark Gap of 3 mm. was used.

DIAGRAM IX



Relation between Spark Potential and Product (Pressure × Spark Length) for uniform Fields in Air, Hydrogen, Sulphide, Sulphur Dioxide, and Electrolytic Oxygen at Low Pressure (Curve due to Carr). A Constant

acetylene. (See also Wolf [22] for sparking in gases other than air.)

(ii) *Temperature Effects on Spark Potentials—Limits of Paschen's Law*

TEMPERATURE EFFECT

It has already been stated that the effect of temperature is included in the density. That is to say, temperature has no intrinsic effect. Paschen's law in its most general form is

Sparking potential=function (number of molecules between the electrodes)

$$=\text{function}\left(\frac{pl}{\mathrm{T}}\right) \quad . \quad . \quad . \quad . \quad b2.3$$

where l=spark length, p=pressure, T=absolute temperature.

For the density (number of molecules per cubic centimetre) is proportional to the ratio $\frac{\text{pressure}}{\text{absolute temperature}}$. The law in this last form has been investigated by Cardani [27] and Guye and Stancescu [15] over a considerable range of temperatures, and they have found the relation to hold within the limits of experimental error.

HIGH PRESSURE LIMIT

The two forms of Paschen's law given above cannot be identical if Boyle's and Charles' laws do not hold. At high pressures and ordinary or low temperatures this is the case. Guye and his collaborators have carried out many investigations for high pressures, using air, carbonic acid, and nitrogen, and they have found that, if the form of Paschen's law given in (*b*) (i), viz. sparking potential=function (density × spark length), is employed, it holds over a much greater range than that for which Boyle's and Charles' laws hold for the gases concerned. Hammershaimb and Mercier [28] support this, and, for spark gaps less than 1 mm.,

[27] Cardani, *Rend. della R. Acc. dei Lincei*, **6**, 44 (1888).

[28] Hammershaimb and Mercier, *Arch. des Sciences*, **3** (1921).

consider the law holds up to about 10 atmospheres pressure. Schumann's [26] results also support Guye's contention.

(iii) *General Shape of the Curve*

LOW PRESSURES

At low pressures, *i.e.* low values of the product, spark length ×pressure, we notice a fall to a minimum for the sparking potential, and a subsequent rise. Paschen's law also ceases to hold under these conditions. The spark discharge is at low pressures modified by two conditions :

(1) There must be a sufficient number of molecules between the plates.
(2) The mean free path must be small compared with the spark length.

Both these conditions must be satisfied by the pressure and spark length, otherwise there is no spark, but only a vacuum glow discharge. The minimum is due to the fact that decrease of pressure, or decrease in the number of molecules, favours ionisation by reason of the greater ionic collision energy, due to longer paths, but opposes it by reason of the fewer molecules which can be hit. At higher pressures the first consideration predominates, at lower pressures the latter. Pressures and spark lengths below this minimum must be considered with discharge tube phenomena, which are outside the scope of this volume.

HIGHER PRESSURES

At higher pressures we notice a falling off of the voltage from proportionality to the spark length (*i.e.* the field strength decreases). We might explain this, both by the increased space charge effects which are present in the heavier discharges, and also by increases in the values of the coefficients of ionisation by collision. At present there is not sufficient evidence to make quantitative statements.

(*d*) Sparking Potentials at High Pressures

(i) *General Form*

When pressures over 10 atmospheres are employed, we begin to find considerable variations from Paschen's law, which increase to such an extent that the law can no longer be considered as at all valid above about 10 atmospheres for air, and above lower pressures, with more complex gases, such as carbonic acid.

On Diagrams X–XIII are given Amyashi's [29] curves for the relation between sparking potential, pressure, and spark length, up to 70 atmospheres, for air, nitrogen, and carbonic acid. These curves are fairly representative, but comparison is difficult owing to the fact that other workers have tended to state their results qualitatively. Guye [30] and his collaborators have examined air and carbonic acid with some care at high pressures. Schumann [26] has studied air and nitrogen, as also have Hammershaimb and Mercier.[28]

The general result is that sparking potentials are too small to satisfy Paschen's law at high pressures. They tend to limiting values for a given spark length as the pressure increases. Schumann [26] observes that at high pressures the actual spark is very bright and voluminous.

The theories put forward to account for these effects will be considered separately, for they are not only intrinsically interesting, but in many cases illustrate the author's experimental results, which are otherwise difficult to comprehend.

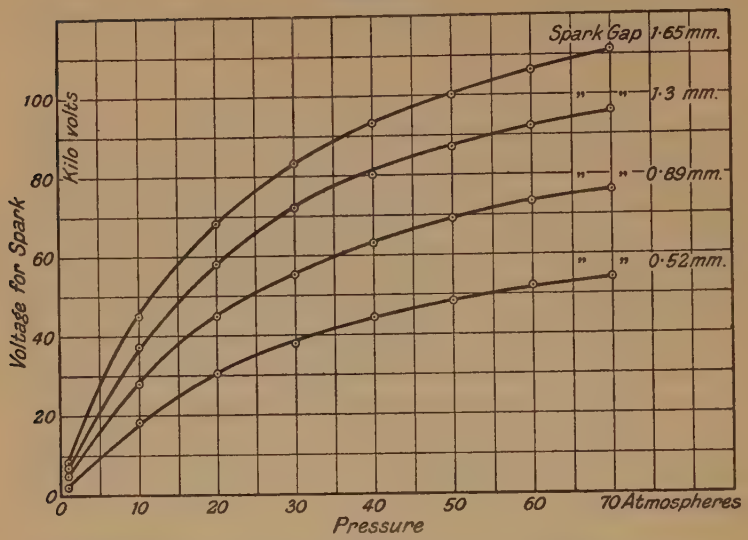
(ii) *Molecular Field Theory*

BOUTY

It was early put forward, and has recently been summarised by Bouty,[31] that an effect exists in gases called by him *dielectric cohesion*. The development of the theory is given in Appendix IV,

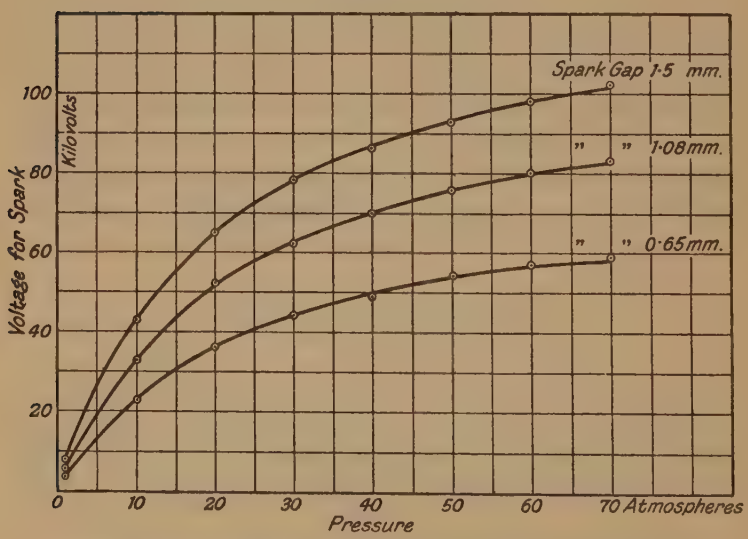
[29] Amyashi, *Ann. der Phys.*, **45**, 3 (1914).
[30] Guye, *Arch. des Sciences* (1922).
[31] Bouty, *Ann. der Phys.*, **16** (1921); *Compt. Rend.*, **17**, 2 (1921).

DIAGRAM X



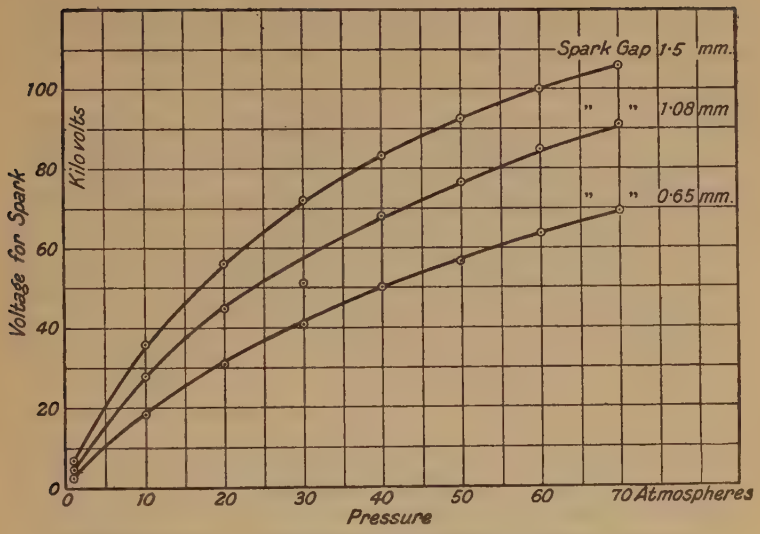
Relation between Spark Potential and Pressure at High Pressures for uniform Fields in Air, according to Amyashi.

DIAGRAM XI



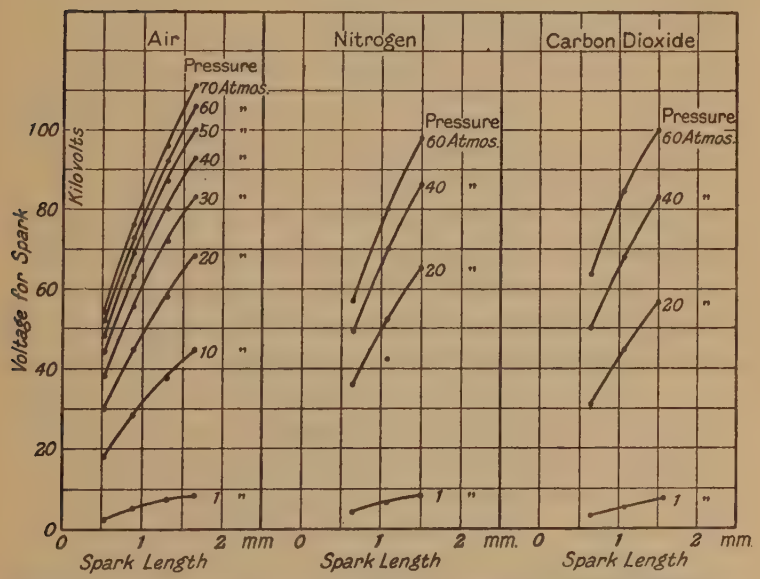
Relation between Spark Potential and Pressure for uniform Fields in Nitrogen at High Pressures, according to Amyashi.

DIAGRAM XII



Relation between Spark Potential and Pressure for uniform Fields in Carbon Dioxide at High Pressures, according to Amyashi.

DIAGRAM XIII



Relation between Spark Potential and Spark Length for uniform Fields in Air, Nitrogen, Carbon Dioxide at High Pressures, according to Amyashi.

and only the details of importance will be given here. Bouty thinks that the molecules of a gas are to some extent dissociated or polarised in an electric field. If the field is strong enough they become ionised. The force which prevents their becoming ionised is called *dielectric cohesion.* The " critical " field in which " dielectric cohesion " breaks down was, he found, at higher pressures and larger distances the same as the ordinary disruptive discharge potential. Some doubt exists as to what exactly he means and whether his theory is not completely false, but nevertheless Guye [30] has considered seriously the aspect of gaseous polarisation.

GUYE

According to the latter, at high pressures this polarisation becomes considerable for strong fields, as we should expect if, as on Bouty's view, the gas is going to be broken down. There is therefore an increase in effective field strength due to the polarisation. Accordingly, sparking occurs at a lower apparent potential, since the molecular polarisation must, other things being equal, be proportional to the density. The correction to Paschen's law can be expressed thus :

$$[A]V = f(\delta d) \text{ (see Appendix IV)} \quad . \quad . \quad b2.4$$

where δ=density, d=spark length, V=spark potential, and [A] is some function of the density.

According to this, the variation with density should be characteristically different from the variation with spark length. In practice (compare Diagrams X–XII with XIII) this is not so. Further, if the molecular fields become large there must be an abnormal increase in the dielectric constant, for if it increases normally, then seeing that it is 1·06 for air at 100 atmospheres, we should expect the field strength could be increased only by 6 per cent. The refractive index is, indeed, largely modified at high pressures, but this does not prove the existence of strong molecular fields. If the dielectric constants of gases were determined at high pressures, it might very possibly be found that this molecular polarisation has some effect, although probably a minor

one. This is a subject for experiment. Bouty's type of breakdown is not very probable.

(iii) *Rudy's* [21] *Theory*

In Diagram XIV curve A shows the spark potential plotted against the product of spark length by density ($\delta \times S$) for spark lengths of 0·3 and 0·2 cm. Owing to Paschen's law the curves for both these lengths are identical, the pressures not being very high. Curve B shows the potential for a spark length of 0·05 cm. plotted in the same way. As the spark length is so small, the pressures for the same value of the product (δS) are high, and so it shows the effect of high pressure on sparking. We see that the spark potential for the high pressures falls off rapidly from the theoretical Paschen's law values. Curve C, on the same diagram, shows ϵ, the difference between the spark potentials for the two curves. It may be represented roughly as having the value

$$\frac{a}{e^{-\frac{b}{S\delta}}-1} \quad . \quad . \quad . \quad . \quad b2.5$$

where δ=density, S=spark length, a and b are constants. Curve D shows the variation of the curve:

$$\epsilon^1=\frac{d\epsilon}{d(S\delta)} \quad . \quad . \quad . \quad . \quad b2.51$$

Curve D resembles the variation of specific heat with temperature at low temperatures.

On this analogy, Rudy suggests that energy is given up by the molecules *in quanta* during collisions. At low pressures, owing to the relatively small number of collisions, the storage of energy is small. At high pressures, owing to the number of collisions, this energy storage becomes large, and the molecules are thereby more easily ionised, as the electrons, having more energy, are more loosely held.

He supports this by pointing out that the refractivity and the absorption and emission spectra of gases are profoundly modified

at high pressures. Absorption is only a half-way house to ionisation. The spark discharge also is heavy at high pressures, indicating excessive ionisation.

He does not give any formula, however, to connect the different effects. His theory is very interesting, and such calculations might be instructive. On this theory the temperature should have a specific effect, apart from its decrease of the density. If one carried the specific heat analogy further, one might expect some relationship such as

$$\frac{\text{sparking voltage}}{\text{some characteristic voltage, } e.g. \text{ ionisation potential}} = \text{function} \frac{\text{density} \times \text{spark length}}{\text{characteristic mass, } e.g. \text{ mol. wt.}} \quad b2.6$$

analogous to the relation—

$$\left.\begin{matrix}\text{specific heat} \times \text{atomic} \\ \text{weight}\end{matrix}\right\} = \text{function} \frac{\text{temperature}}{\text{temperature corresponding to oscillation limit}},$$

which is the same for all substances.

(iv) *Schumann's Theory* [32]

Schumann supposes that at very high pressures the electrons ejected in the course of ionisation unite with molecules, and, in the high field strengths which are necessary to give discharge, cause a greater ionisation than is provided for in Paschen's law. He has worked out on these lines a complicated theory which is difficult to test. It appears, however, that it suffers from the difficulty that the effect would be proportional to the density, which, as we have seen, would render it not completely adequate.

(v) *Hammershaimb and Mercier's Theory* [28]

These two authors suggest that the variations from Paschen's law at high pressures are due to inequality in the space charges, owing to the relatively lower mobilities of the ions. The field is rendered non-uniform, and the spark facilitated by local flux concentrations. They are, however, rather vague, and put forward no calculations and not much specific evidence.

[32] Schumann, *Archiv für Electrotechnik*, **12** (1924).

DIAGRAM XIV

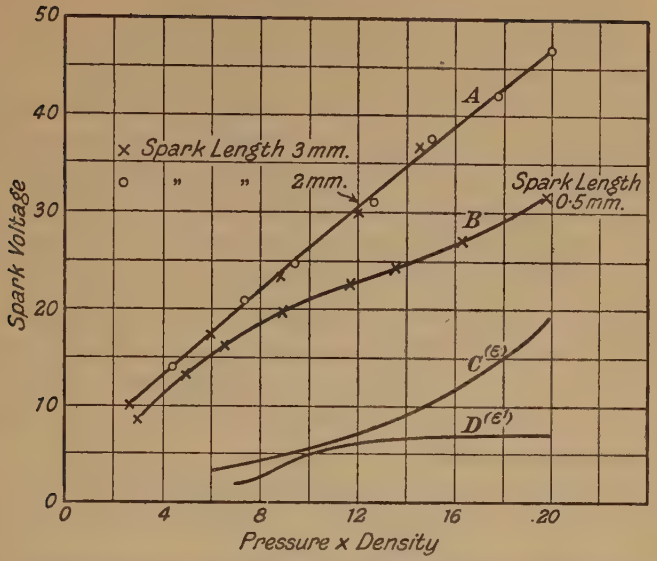
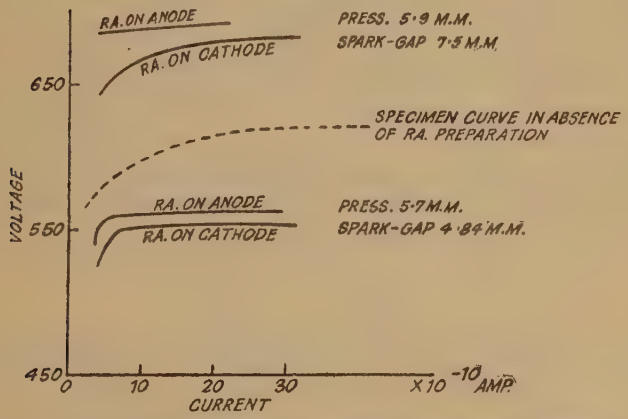


Diagram given by Rudy showing the Variations from Paschen's Law at High Pressures.

Curve A obeys Paschen's Law (Spark Length 2 × 3 mm. are plotted).
" B, owing to High Pressures used (since Spark Length is small), does not obey Paschen's Law.
" C represents the difference between A and B, *i.e.* $\epsilon = V_A - VB$.
" D is first derivative at Curve C, *i.e.* $\epsilon = \frac{d\epsilon'}{d(S\delta)}$ where S = Spark Length, δ = Density.

DIAGRAM XV



Diagrams given by Reiche showing Effect of Radioactive Material on the Currents between parallel Plates in the neighbourhood of Sparking Voltage.

(e) Other Effects

(i) *Initial Ionisation*

The results of earlier experimenters in the field of spark potentials, such as Liebig [19] and Baille,[3] often showed large experimental variations, and their values were higher than those of later experimenters. This was due to the fact that they neglected to ensure the presence of a certain amount of initial ionisation. No discharge will start without a few ions at the beginning, but, since there are always a few ions present in any sample of air, a spark will pass between parallel plates whether artificial ionisation is supplied or not, provided the potential is maintained long enough for the atmospheric ions to multiply themselves sufficiently.

In practice, however, a steadily and sometimes rapidly rising source of potential is provided: so that, if the initial ionisation is not supplied, the potential will rise to a false value, owing to the lag. If, however, A.C. is used, once a series of sparks has been started there is always sufficient residual ionisation from the previous cycle to start the discharge.

Reiche [33] has studied the effect of initial ionisation by measuring, very carefully, the variation of current, as the potential is increased from below to above the spark potential. He finds that, at ordinary pressures, the current is discontinuous at sparking, showing that the discharge develops practically instantaneously. For low pressures, however, the current curve shows a distinct bend or "knee." If a radium preparation is placed upon an electrode, the bend is made much shorter and the corresponding voltages are reduced. Radioactive preparations on the cathode produce a greater lowering than on the anode. Some representative curves are shown in Diagram XV.

(ii) *Nature of Surface*

Discontinuities on the surface of electrodes give low sparking potentials, owing to local discharges. In the same way dirt and impurities give a lowering owing to local flux concentrations.

[33] Reiche, *Ann. der Phys.*, 57 (1917).

Moisture and oil [34] also lower the spark voltages, the former more than the latter, probably owing to the high dielectric constant of water. The amount of this lowering is, however, not very large with uniform fields, although considerable with non-uniform fields.

In making accurate measurements one of the chief difficulties encountered is with edge effects. Most experimenters, instead of parallel plates, have used large spheres close together, or else belled out the edges of their plate electrodes. Carr,[13] whose experiments were very carefully carried out, embedded his electrodes in ebonite. He also polished the electrodes very carefully before making an experiment.

Some recent results on spheres may be mentioned here which were communicated to the E.R.A. by the British Thomson-Houston Co., Ltd., Rugby. Irregular sparkover results were obtained both with a mirror surface and with a rough or dirty surface. The most consistent results occurred when the spheres were of brass and had merely been polished with an ordinary metal polish or similarly treated. Microscopical examination showed that in the latter case the surface was covered with scratches. It is also necessary to make a few preliminary discharges across any spark gap before taking readings.

It is possible, therefore, that Carr's results might have been more consistent had he taken less care in the preparation of the surface. The matter, however, is still in the empirical stage, since the precise physical effect of polishing, etc., is only just becoming clear.

These effects produce the greatest errors at lower pressures and distances, and may be partly responsible for some of the considerable divergences shown, under those conditions, by the results of different experimenters.

(iii) *Frequency*

Leontiewa [36] has found that, for high frequency oscillations, the discharge voltage is a linear function of the spark length.

[34] Peek, *Dielectric Phenomena*, ch. iv.
[36] Leontiewa, *Phys. Zeit.*, **23** (1922).

(iv) *Mixtures of Gases*

According to Hayashi,[135] the spark potential of a mixture of gases is the mean of its components. Roughly, this is to be expected, but for very accurate measurements with accurate voltage control, the weaker (*i.e.* the one with lower ionisation potential) would probably have a preponderating effect. This author also suggests that contamination due to chemical changes has an important effect on sparking, particularly at high pressures, the compounds formed lowering the voltage. There does not seem much evidence for this, and corona would present a difficulty. Ozone is formed here, and the more ozone the stronger should be the discharge on this theory. The corona would on this account tend to instability, and one would expect a large quantity of ozone to be formed before equilibrium occurred. Actually, only very small quantities of any gas compound are formed in a discharge of normal duration.

III. NON-UNIFORM FIELDS

(*a*) General

(i) *Resemblances between Sparks*

The phenomena in fields where the lines of force are not parallel are very complicated. In the first place, other discharges, *e.g.* the silent glow or brush discharges, may occur first, and their presence or absence has been put forward by many experimenters in explanation of some of the anomalous results obtained. In the second place, it is more difficult to calculate the electrostatic field of the different systems, and so far it has been impossible to observe the field during discharge. Nor can we feel at all easy about any assumptions that we make with reference to the field in discharge. Accordingly, in sparking in non-uniform fields, we must at present approach the matter more or less empirically, and treat each case on its merits.

One does observe, however, that there is a certain similarity between the experimental laws for different arrangements. As

[135] Hayashi, *Ann. der Phys.*, **45**, 3 (1914).

we shall see, concentric cylinders, parallel wires, and the needle gap tend to give more or less linear relations between the voltage and spark length. From a general consideration it would appear that the fields during the passage of a spark tend to resemble each other much more than do the corresponding electrostatic fields, *i.e.* the spark tends to reduce a field to a common form. Of course this is true only in a vague sort of way, but it does obtain some support from other phenomena.

The sparkover voltage may be expected to show less change with electrode shape than the corona voltage on the basis of the considerations developed in I (*c*), as a simple example will show. The continuity condition,

$$\int_0^l \alpha e^{\int_0^x \beta - \alpha dx} dx = 1 \quad . \quad . \quad . \quad b3.1$$

should hold for the spark as well as corona.

If we put $\beta = k\alpha$, we have

$$\int_0^l \alpha dx = \frac{\log k}{k-1} \quad . \quad . \quad . \quad . \quad b3.2$$

If we assume $\alpha = \lambda(X - X_0)$, a form which was found fairly satisfactory in uniform fields, we have

$$\int_0^l (X - X_0) dx = \frac{\log k}{\lambda(k-1)},$$

where X = field.

$$\therefore \quad V - lX_0 = \frac{\log k}{\lambda(k-1)} \quad . \quad . \quad . \quad b3.3$$

i.e. the sparkover voltage should show the same variation with distance as for a uniform field. This would make the sparkover voltage independent of the electrode shape, which is, of course, untrue. Nevertheless, apart from the effect of the space charge, the above calculation shows that electrode shape should have less effect upon the sparkover voltage than upon the corona voltage, or than the effect upon the electrostatic field would suggest.

The above remarks are also expressed in the theory already mentioned that the continuity condition (*b*3.1) holds right up to sparkover, but that the electric field is modified by the discharge. If corona does not precede sparkover, it is clear that the modifica-

tion is slight, and so *b*3·1 will yield the same result both for corona and sparkover.

(ii) *Change of Glow and Brush into Spark*

In the case of the wire and cylinder, spheres, parallel wires, and similar arrangements, by varying the curvature of the electrodes relative to the spacing we find that discharge phenomena can be divided into two types : those where the spark only occurs and those where corona precedes sparking.

In the former type, the spark voltage and gradient obey the extrapolated law which the corona voltage obeys when, as in the second type, corona precedes spark. When corona happens first, the spark voltage tends, in many cases, to obey a linear law (as far as spark length is concerned).

Now, in corona, we have partial breakdown, *i.e.* ionisation by collision takes place over a limited distance and in the more intense parts of the field. The spark, we may say, does not occur, because this distance is not a sufficiently large fraction of the total distance between the electrodes. But when the dimensions of the apparatus are such that the thickness of the corona layer is sufficiently great, the discharge jumps the rest of the distance, becoming a spark in consequence of the large consequent ionisation. The voltage, however, need be sufficient only to start off the corona, the rest following automatically.

The above may be expressed mathematically by saying, as in the Introduction, III (*c*), that the continuity condition (*b*3.1 above),

$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx = 1,$$

holds for corona. With increasing voltage and with increasing values of the ratio $\frac{\text{corona thickness}}{\text{total gap length}}$, it becomes more and more difficult for the field to adjust itself by means of the space charge, so that the condition holds with a finite current through the gas. Finally, as either the voltage is increased or the ratio curvature of electrode/gap length is decreased, the condition becomes impossible. The current rises unstably and a spark passes.

These considerations will be recapitulated in Part III on Corona.

(iii) *The Voltage Drop*

We may here conveniently consider the voltage drop which occurs on passage of a spark. The spark produces the greatest fall of potential energy. It will therefore, on the principle of stability requiring minimum potential energy, be the type of discharge to which other types tend, and, further, it is not unreasonable to suppose that for different arrangements it will lead to a greater uniformity than either the electrostatic fields or the fields of other discharges.

(iv) *Asymmetry of Positive and Negative Sparkover*

It was shown in the Introduction, III (*d*), that, owing to the difference between the efficiency of the positive and negative ions, the discharge potential depends in general upon the direction of the field. This applies to sparkover as to other discharges, but the difference is usually less owing to the tendency for the field during a spark discharge to resemble a uniform field. Again, in very non-uniform fields where the difference is greatest, corona occurs first, and attention has been directed away from the spark discharge.

The general principle given in the Introduction holds here, viz. that the negative discharge takes place at a lower potential for higher values of $\frac{X}{p}$ (X=field strength, p=pressure), and that for lower values the positive discharge occurs more easily.

(v) *Homologous Fields*

Guye and Mercier [30] have observed that homologous fields give results similar to those for uniform fields. In such fields the representative spark path would be along a straight line through the centre of homology. If spark fields tend to possess a general resemblance, we should expect them to tend to uniformity, for the uniform field is a particular case of the homologous field, viz. one where the centre of homology is at infinity.

(*b*) Spherical Electrodes

(i) *The Electrostatic Field*

The electrostatic field between two equal spheres leads to complicated formulæ if a complete solution is desired. Such a solution may be found in Russell's *Alternating Currents*, chap. viii. Here only the formula for the maximum gradient, *i.e.* the stress at the spherical surface, will be quoted.

If R_0=surface stress, S=spacing, a=common radius of spheres, and V_1 and V_2 are their potentials, then

$$R_0=\frac{V_1-V_2}{x}f_1+2\frac{V_2}{x}(f_1-f), \quad . \quad . \quad b4.1$$

where f and f_1 are functions of $\frac{x}{a}$ (Appendix V). (These functions have been calculated for values of $\frac{x}{a}$ from 0 to 2, and are given in the tables attached to Ref. L/T16 (see also Appendix V).)

In practice, the spheres are either at equal and opposite potentials, or one is earthed. In these cases respectively *b*4.1 reduces to

$$R_0=\frac{V}{x}f \quad \text{and} \quad R_0=\frac{V}{x}f_1 \quad . \quad . \quad . \quad b4.2$$

V now represents the potential difference between them.

The complete expression for the force at any point of the axis of the arrangements is of a complicated nature, and so far no convenient method of handling it has been found. Accordingly, an approximate solution put forward by Dean [138] is given :

$$R=\frac{V}{x}\left\{2x^2\frac{\left[x^2(f+1)+4\left(\frac{x}{2}-y\right)^2(f-1)\right]}{\left[x^2(f+1)-4\left(\frac{x}{2}-y\right)^2(f-1)\right]^2}\right\} \quad . \quad b4.3$$

R=gradient at a point on the axis distant y from the surface of one sphere.

[138] Dean, *G.E. Review* (March 1913).

(ii) *General Nature of Sparkover for Spheres*

At a certain voltage for a given setting, the sphere gap breaks down and a spark forms first.

The variation of sparkover voltage with gap setting or spacing is well known. A few curves from Peek [42] and Klemm [40] are given on Diagrams XVI and XIX.

If, however, we calculate the surface gradient for breakdown by means of the formulæ given in (i), we obtain much more interesting curves. On Diagrams XVII and XX (*a*) are shown some curves of gradient against $\frac{x}{a}=\frac{\text{spacing}}{\text{radius}}$ from Klemm,[40] Baille,[9] Paschen,[10] Heydweiller,[41] and *A.I.E.E.* calibrations.

We observe that the ratio $\frac{x}{a}$ is the best independent variable: plotted against x, the curves would not be nearly so directly comparable. Further, it is noticed that the curves may roughly be divided into three parts:

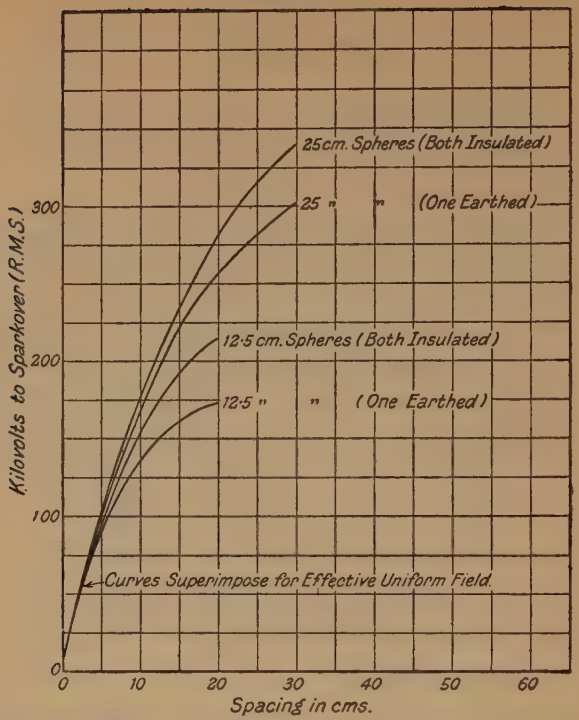
1. A portion extending from 0 to $\frac{x}{a}=0{\cdot}3$ or $0{\cdot}4$ for larger spheres and $\frac{x}{a}=0{\cdot}5$ for very small spheres. Over this range it may be noted that the voltage *versus* spacing curves for different spheres run together, *i.e.* the sphere gap acts as if it were a uniform field. This portion may therefore be known as the effective uniform field.

2. Between the minimum mentioned and $\frac{x}{a}=2$ (about), the gradient curves, after passing the minimum, rise slowly with $\frac{x}{a}$, showing a curvature towards the origin.

3. When $\frac{x}{a}>2$, the gradient rises steadily but very slowly in a practically linear manner.

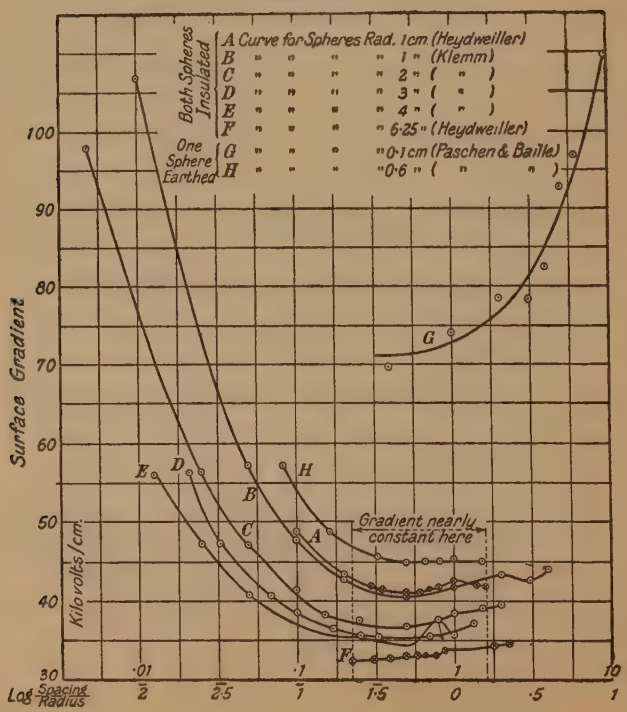
[42] Peek, *Dielectric Phenomena*, p. 92.
[40] Klemm, *Archiv für Electrotechnik*, **12** (1923).
[41] Heydweiller, *Ann. de Phys. und Chem.*, **48** (1893).

DIAGRAM XVI



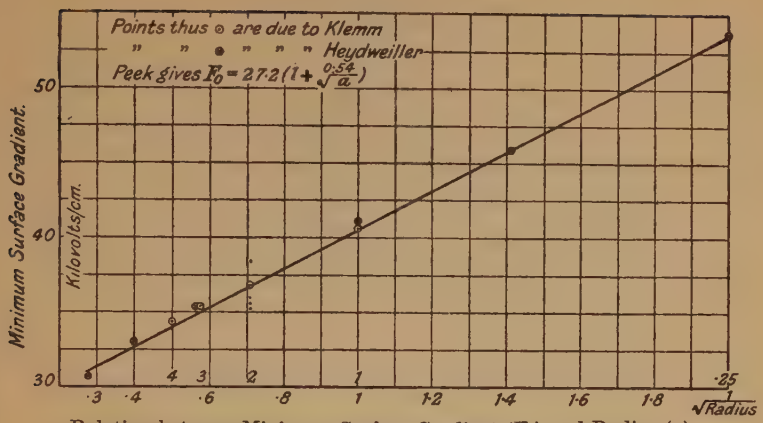
Sparking Voltages for Spheres at different Spacings (Peek).

DIAGRAM XVII



Relation between Surface Gradient (F_0) and Ratio $\frac{\text{Spacing}}{\text{Radius}}$ for different-sized Spheres Radius

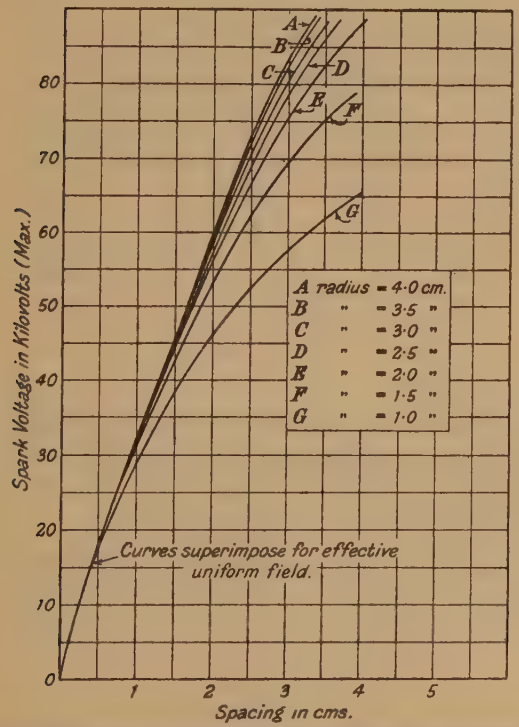
DIAGRAM XVIII



Relation between Minimum Surface Gradient (F_0) and Radius (a).

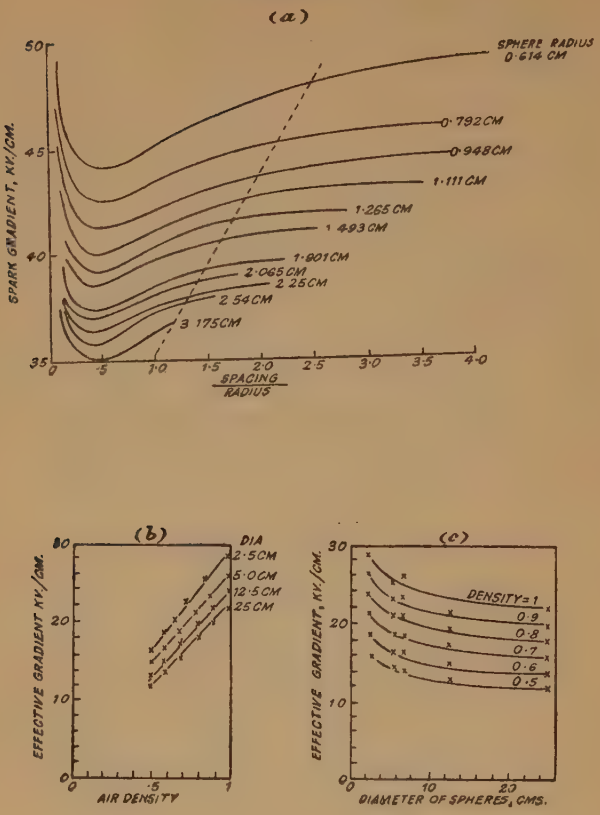
Test of Formula $F_0 = A + \frac{B}{\sqrt{a}}$. A and B are Constants.

DIAGRAM XIX



Relation between Spark Voltage and Spacing from Results by Klemm.

DIAGRAM XX



(d)

SPHERE RADIUS IN CM.	0·25	0·5	1	3·125	6·25	12·5	25
SURFACE GRADIENT, OBSERVED	54·6	47·2	41·5	36·2	33·4	31·3	30·2
SURFACE GRADIENT, CALCULATED	55·6	47·5	41·5	35·4	33·2	31·4	30·2

Comparison of Russell's Correction (Surface Gradient proportional to Density) with Experiment (Heydweiller's Figures).

(iii) *Experimental Laws*

1. Over the effective uniform field, the laws which apply to uniform field sparking apply here. As they have already been dealt with they will receive no further mention.

2. Over the range 2 in the previous paragraph, the sparkover voltage follows the same law (extrapolated) which the corona voltage follows when it occurs, *i.e.* the sparkover curve for this portion is continuous with the corona curve. This, as has already been mentioned, is a general principle in non-uniform fields.

3. Although corona is not visible until $\frac{x}{a}>4$, the sparkover voltage begins slightly to follow a different law after $\frac{x}{a}>2$. The gradient appears to rise linearly, but the variation is small. Later on, however, for larger spacings, the sparkover voltage begins to follow a linear law also. At these spacings, however, $\frac{x}{a}$ is so large that the field approaches that between points.

Range 2 is the most interesting, as over this range the sphere gap is used as a voltmeter. The following law appears to be in agreement :—

For a given value of $\frac{x}{a}$,

$$R=A+\frac{B}{\sqrt{a}} \quad . \quad . \quad . \quad . \quad . \quad b4.4$$

A and B are functions of $\frac{x}{a}$, and A appears to tend to a value between 30 and 31 *k*V/cm. for values of $\frac{x}{a}=2$.

It is possible to represent with fair accuracy (particularly with small spheres) the variation of R (and therefore the sparkover voltage) with spacing and radius by a single formula (see Ref. L/T16).

(iv) *Theories of the Sphere Gap*

It must first be pointed out that for the very large spacings where corona occurs separately, there is no theory except Peek's

experimental law that the sparkover voltage is a linear function of the spacing.

For smaller values of $\frac{\text{spacing}}{\text{radius}}$ the theories applicable must be theories of corona, since corona laws apply. Accordingly, Peek and Townsend's theories, which are theories of corona, will not be mentioned here, but will be dealt with in the section dealing with corona.

According to Russell,[37] the surface stress at sparkover is constant (for spacings>2 mm.) for a given sphere, and follows the law determined by the equation

$$R=27{\cdot}2+\frac{14{\cdot}7}{\sqrt{a}}kV/cm. \quad . \quad . \quad . \quad b4.5$$

In the case of insulated spheres he ignores the variation with $\frac{x}{a}$ which we have observed to take place. In the case of one sphere earthed, he assumes that the actual voltage of the earthed sphere is not zero but 43 per cent. of the total applied volts below zero. Thus he uses formula *b*4.1 to calculate R, making

$$V_2=0{\cdot}43(V_1-V_2).$$

In the case of the *A.I.E.E.* results a moderately constant gradient is so obtained, but the divergences from the mean are comparatively large. The theory does not agree very well with the results obtained by other experimenters, and in any case the assumption would require some definite evidence.

Yoshida [48] has propounded a somewhat similar theory. He states that the applied voltage is not the effective voltage, but is reduced by a sort of contact potential drop. Further, the gradient is not constant but rises linearly with $\sqrt{\frac{x}{a}}$, owing to the disturbing effect of extraneous bodies. However, actually the gradient does not rise in such a manner, nor are its variations such as might be ascribed to stray effects, although naturally they enter into the matter. The contact potential drop is also a considerable assumption.

[37] Russell, *J.I.E.E.*, **37** (1920), supplement, part 2, pp. 2 and 3.
[48] Yoshida, *Kyoto College of Science*, Mem. 5 (1921).

(v) *Effect of Density of Air*

Temperature and pressure affect only the sparkover voltage in so far as they alter the density of the air.

It will be remembered that the equation

$$R=A+\frac{B}{\sqrt{a}}\left(\frac{x}{a}\text{ given}\right)$$

was an experimental law for the arrangement. We may now apply Townsend's general discharge theorem in the form given by the formula *a*3.2.

The parameter here is the radius, so that if δ=density,

$$\frac{R}{\delta}=A+\frac{B}{\sqrt{a\delta}} \quad . \quad . \quad . \quad . \quad b4.6$$

represents the effect of density. The formula has not been tested in this form. Generally, the average gradient over a range of $\frac{x}{a}$ has been taken, and this average has been found to follow the above law closely [43] (see Diagram XX (*b*) and (*c*)). This follows if *b*4.6 holds for

$$\frac{\Sigma R}{\delta}=\Sigma A+\frac{\Sigma B}{\sqrt{a\delta}},$$

but $R_{mean}=\frac{\Sigma R}{n}$, n=number taken, and similarly for A_{mean} and B_{mean}.

Whence

$$\frac{R_{mean}}{\delta}=A_{mean}+\frac{B_{mean}}{\sqrt{a\delta}} \quad . \quad . \quad . \quad b4.7$$

the n's cancelling out. On Diagram XXI, V, the voltage, has been plotted as a function of δa. From *b*4.6 we should expect that

$$V=\left(\frac{A}{f}\frac{x}{a}\right)\delta a+\left(\frac{B}{f}\frac{x}{a}\right)\sqrt{\delta a}.$$

For a given value of $\frac{x}{a}$ this is

$$V=A_1\delta a+B_1\sqrt{\delta a} \quad . \quad . \quad . \quad b4.8$$

---

[43] Peek, *Dielectric Phenomena*, p. 93.

Some results for values of $\frac{x}{a}=1$ and $\frac{x}{a}=1{\cdot}07$ from Peek, Klemm, and Heydweiller are given, and it is observed that they all give V as a linear function of δa. This is to be expected, owing to the small variations of δ considered. It is interesting to note that, although the values of $\frac{x}{a}$ are nearly identical for the two sets, they are not quite, and this difference is reflected in the slight difference between the two lines.

Russell [37] states that the voltage is proportional to the density, and for small variations, such as are met with in ordinary practice, this is true. (For work on the sphere gap see also Chubb and Fortescue,[44] Peek [45, 46] and Townsend.[38])

(vi) *Sphere and Plane*

Although the electrostatic field of the sphere and plane is identical with that of two spheres, their sparking potentials are naturally different, owing to the ionisation cut off. According to Hammershaimb and Mercier,[28] at ordinary pressures the arrangement gives results intermediate between those for two planes and those for two spheres (this seems a general rule for dissimilar electrodes). At high pressures, strangely enough, although for spacings up to 1 mm. hemispheres of small diameter give spark potentials higher than for plane electrodes and spherical caps, for greater distances the position is reversed.

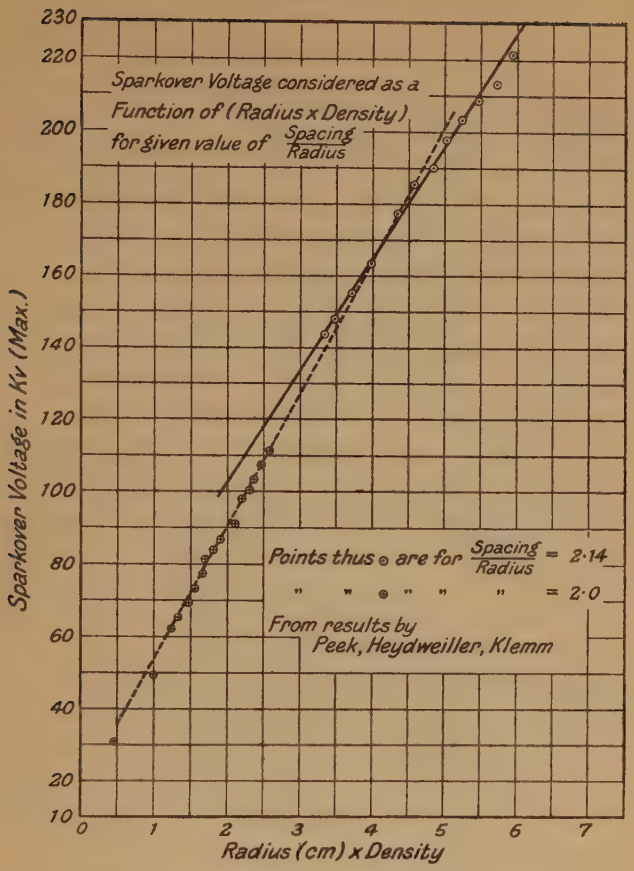
(*c*) Wire and Cylinder (or two Concentric Cylinders)

(i) *Presence of Corona*

With this arrangement corona very often occurs at voltages considerably less than are necessary to produce a spark. Further, the development of corona before the spark marks a discontinuity in the sparking laws. Accordingly, it is best to divide the phenomena into two classes: those in which a spark only is obtained and those in which corona is formed first.

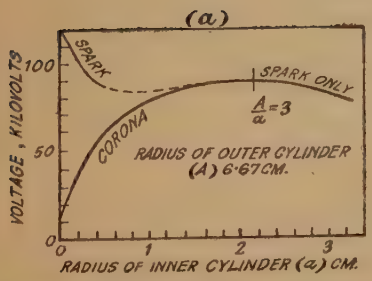
[44] Chubb and Fortescue, *A.I.E.E.* (Feb. 1913).
[45] Peek, *A.I.E.E.* (Feb. 1913). [46] Peek, *G.E.R.* (May 1913).
[38] Townsend, *Electricity in Gases*, p. 340.

DIAGRAM XXI

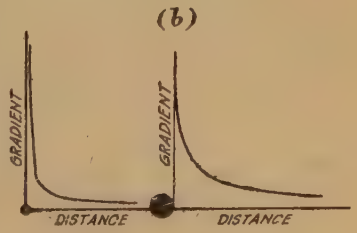


Sparkover Voltage and Density.

DIAGRAM XXII



Relation between Spark and Corona Voltages and Inner Cylinder Radius for Concentric Cylinders.



Variation of Gradient with Distance from Surface for small and large Vites in a Concentric Cylinder.

(ii) *Forms where Spark only occurs*

When the ratio of the diameters of the wire and the cylinder, viz. $\frac{\text{diameter of wire}}{\text{diameter of cylinder}}$, exceeds a certain value (about 3) only a spark occurs, and the spark potential follows the same law (extrapolated) which the corona voltage does where the corona occurs first. This law is

$$\frac{V}{a \log \frac{A}{a}} = g_s = g_0\left(1+\frac{0{\cdot}3}{\sqrt{a}}\right)^{49} \quad . \quad . \quad b5.1$$

where V=spark potential, g_s=gradient at wire surface, g_0=constant (about 30 *k*V/cm.), A=radius of cylinder, a=radius of wire. Since this law is primarily a corona law, it will be discussed fully in the section on corona. The theory is that partial sparking takes place from the surface of the wire to points where the field falls below g_0 (a distance $0{\cdot}3\sqrt{a}$ from the surface). When this distance $0{\cdot}3\sqrt{a}$ is sufficiently large compared with the distance between the surfaces A and a, *i.e.* when the ratio of wire diameter to cylinder is sufficiently large, the sparking will jump the rest of the gap and become a full spark. The exact point at which corona forms independently is difficult to determine, and a discussion of the conditions will be deferred to the section on Corona. In Diagram XXII (*a*) is given a curve for sparking potential against wire radius due to Peek.

As we see from the formula in this case, the sparking gradient g_s is independent of the radius of the outer cylinder.[50]

As the ratio of the diameters approaches unity, the phenomena naturally approach more to uniform field sparking.

From the general discharge theorem (see *a*3.2) the effect of density of the gas can be included by writing the formula in the form

$$\frac{g_s}{\delta} = g_0\left(1+\frac{0{\cdot}3}{\sqrt{a\delta}}\right)^{51} \quad . \quad . \quad . \quad b5.2$$

where δ=density.

[49] Peek, *Dielectric Phenomena*, ch. iv.

[50] Gaugain, *Ann. de Chim. et de Phys.*, 418 (1860).

[51] Peek, *Dielectric Phenomena*, p. 57; also Watson, *Electrician* (11th Feb. 1910).

(iii) *Forms where Corona occurs also*

In cases where corona precedes spark, the voltage appears to have a linear relation with the distance between the surfaces. For an outer cylinder of given diameter this would give a linear relation between the voltage and wire diameter. This is shown in Diagram XXII (*a*).

In Diagram XXII (*b*) we see the gradient variation for thick and thin wires compared. For a thin wire the gradient is nearly uniform for a greater distance than for the thicker wire. Accordingly, the linear relation of the voltage for relatively thin wires might be expected, since the field resembles more closely a uniform field.

Schumann [47] has worked out a complete theory of spark potentials and currents between wire and cylinder, but as the results do not show very close agreement, his theory is still mainly of academic interest.

(*d*) Parallel Wires

In most cases, with parallel wires corona appears before the spark, so that the effects will be divided according to whether corona appears or not, as was done in the case of the wire and cylinder.

(i) *Forms where only Spark occurs*

When the wires are sufficiently close compared with their thickness, the spark only occurs. As in the case of the wire and cylinder, the spark in this case obeys the extrapolated law of corona for the arrangement. This law will be discussed more fully in the section on Corona, but it may be said here that the theory is similar to the previous case. Partial discharge takes place to where the gradient falls below a certain value and leads to the law

$$g_s = g_0\left(1+\frac{0{\cdot}3}{\sqrt{a}}\right)^{52} \quad . \quad . \quad . \quad b6.1$$

---

[47] Schumann, *Archiv für Electrotechnik*, **12** (1923); and also *Electrische Durchbruchfeldstarke von Gasen* (Berlin, 1923).

[52] Peek, *Dielectric Phenomena*, p. 82.

where a is the radius, g_s the gradient at the surface of the wire, g_0 is a constant of value about 30 kV/cm. The voltage is given in terms of the gradient g_s by the formula

$$g_s = \frac{V\sqrt{\frac{S}{2a}-1}}{a\left(\frac{S}{2a}-1\right)\cosh^{-1}\frac{S}{2a}} \quad . \quad . \quad . \quad b6.2$$

where S=spacing.

The critical value of the ratio $\frac{S}{a}$, above which corona forms, is, according to Peek's experiments, 30, but the exact value is difficult to determine.

As in the previous case, the surface gradient depends only on the diameter of the conductor and not upon the spacing.

By applying the general discharge theorem as before, we obtain for the effect of the air density δ,

$$\frac{g_s}{\delta} = g_0\left(1+\frac{0{\cdot}3}{\sqrt{a\delta}}\right)^{53} \quad . \quad . \quad . \quad b6.3$$

When the spacing becomes very small the above laws do not hold so accurately, as they are obtained on the assumption that the field does not differ much from that for wire and cylinder, which ceases to be true for small values of $\frac{S}{a}$.

Curves of voltage and gradient against spacing are given in Diagram XXIII (a) and (b).

(ii) *Forms where Corona occurs*

For spacings at which corona occurs the spark gradient, and also the spark voltage, appear to have a linear relation with the spacing for wires of a given diameter. We can obtain a formula for the spark gradient on this assumption. At $\frac{S}{a}=30$ the gradient is given, as we have seen, by the equation

$$g_s = g_0\left(1+\frac{0{\cdot}3}{\sqrt{a}}\right).$$

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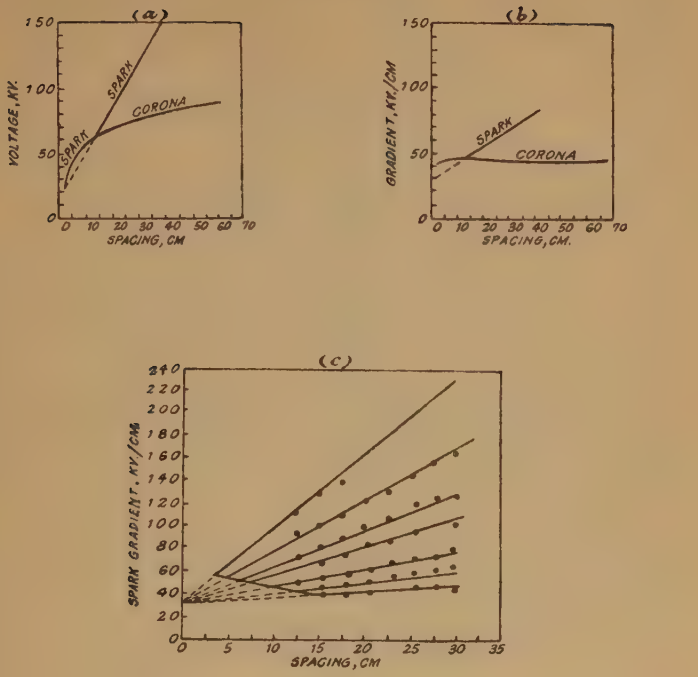
[53] Peek, *Dielectric Phenomena*, p. 57.

DIAGRAM XXIII



Sparking between Parallel Lines.

Further, all the lines of gradient appear to cut the gradient axis where $g=g_0$. They thus pass through the two points g_0, 0; g'_s, $30a$. The equation is therefore

$$\frac{g'_s-g_0}{30a}=\frac{g_s-g_0}{S},$$

i.e.

$$\frac{0{\cdot}3g_0}{30a\sqrt{a}}=\frac{g_s-g_0}{S},$$

i.e.

$$g_s=g_0\left(1+\frac{0{\cdot}3S}{30a\sqrt{a}}\right)=g_0\left(1+\frac{S}{100a\sqrt{a}}\right)^{54} \quad . \quad b6.4$$

This gives the variation of spark gradient with both spacing S and radius a. Curves illustrating this theory are given in Diagram XXIII (*c*) and (*d*), the points being measured and the curves calculated.

In Diagram XXIII (*b*) are shown curves of voltage *versus* radius, the curves being calculated on the above theory and the points measured.

No theory has been put forward to account for the non-appearance of corona below $\frac{S}{a}=30$, and this point will be discussed in the section on Corona. Further, there does not seem any particular reason why all the lines should cut the gradient axis at $g_s=30$ except that, as we have pointed out before, the field may approximate to a uniform field as far as the actual spark path is concerned, which, for atmospheric pressures, can be approximated to a straight line cutting the gradient axis at $g_s=30$.

(*e*) Sparks from Points

The effect of frequency upon sparking at points is very considerable, and the general phenomena connected with frequency and sparking will be dealt with under (*f*) on p. 80. In the present chapter it is assumed that, effectively, spark lag has been eliminated.

[54] Peek, *Dielectric Phenomena*, p. 82.

(i) *Needle Gap*

The spark gap between two needles has been used to determine crest voltages, and standards have been laid down. It is not advisable to use the gap as a voltmeter above 30 *k*V or below 10 *k*V. The A.I.E.E. have recommended the use of 00 sewing needles. The E.R.A. recommend No. 17 English needles, the equivalent in this country (see E.R.A., Ref. L/S2, Appendix V). More recently it has been proposed to abandon needles in favour of small spheres (see E.R.A., Ref. L/T16).

It appears that, provided the gap is fairly long, the sparking voltage is largely independent of the degree of sharpness of the needle points, *e.g.* the needles after being used as arc electrodes do not give very different results.[55] This we should expect as, owing to the very limited range of the intense surface gradient, relatively small changes such as those considered above would not be important.

According to Peek,[56] the voltage bears a linear relation to the spark gap (some of his results are given in Diagram XXIV). This we see is a characteristic of fields which are uniform for considerable distances, with sudden increases at the electrodes (*cf.* parallel wires, wire, and cylinder).

The spark voltage for small changes is proportional to the density of the gas (see Ref. L/S2, Appendix V), but for large changes this law no longer holds.

Wright [59] has measured the spark lengths (relative to an air gap) of the needle gap in different gases, and comes to the conclusion that a high molecular weight decreases the spark length. He does not appear to have allowed for possible spark lag in his comparison gap ; for a fast gap in parallel can mask the effect of a slow gap, which may have a lower actual voltage, as Peek has observed.

The effect of humidity is dealt with under (*g*) on p. 88.

[55] Information communicated to E.R.A.
[56] Peek, *Dielectric Phenomena*, p. 87.
[59] Wright, *J.C.S.*, iii (1917).

(ii) *Single Point* [57, 58]

When a sharp point is raised to a certain potential, a small current passes through the gas and at first no luminous effect is produced. When the potential is increased the current increases and a glow appears, which may subsequently change into a spark. The spark potential is higher the blunter the point, and there is also less difference between the positive and negative discharges. At ordinary pressures the potential for a positive discharge is less than for a negative discharge, but if the pressure is reduced the reverse becomes true. The spark potential is largely determined by the nature of the point, but the distance from other bodies has an important, if smaller, effect.

According to Wolcott,[60] the sparking potential between a positive point and a plane is only half that for a negative point, the effect showing reversal with decrease of pressure. If a discontinuous dielectric surface is interposed, the negative point sparking is lowered to the positive value. There is no lowering when the charge on the surface is conducted away or screened. He thinks that a brush discharge forms first, and supplies the ions for the spark. When a dielectric surface is interposed it becomes charged, and is capable of forming brushes at its roughnesses. Accordingly, brushes of both signs can be formed, so that there is no difference as to whether the point is positive or negative. If the charge is screened or conducted away the effect naturally vanishes.

Sægusa [62] finds that the higher the dielectric constant of the interposed material the lower the spark potential, as is to be expected from the greater flux concentration.

Zeleny [61] gives the relation between sparking voltage V and radius r for liquid points (*i.e.* liquid drops) as

$$V = a\sqrt{r} + b, \quad \text{or} \quad V = a\sqrt{r+b} \quad . \quad . \quad b7.1$$

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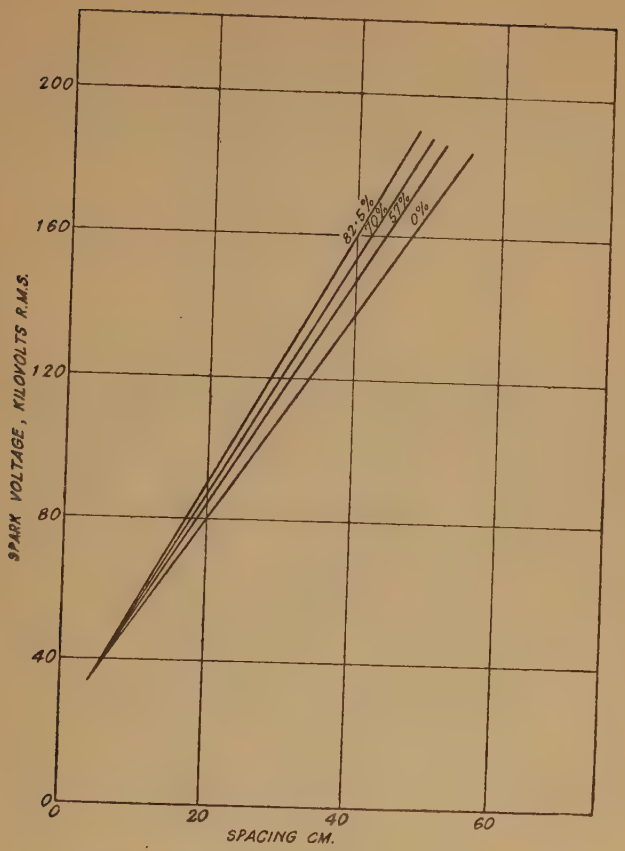
[57] Zeleny, *Phys. Rev.*, **25** (1907).
[58] Gorton and Warburg, *Ann. der Physik*, **18** (1905).
[60] Wolcott, *Phys. Rev.*, **12** (1918).
[62] Sægusa, *Tohoku Univ. Science Reports*, **9** (1920).
[61] Zeleny, *Phys. Rev.*, **16** (1920).

DIAGRAM XXIV



Needle-gap Curves for different Humidities (Peek).

where a and b are constants. This holds only over a limited range.

According to the general sparking theorem, the spark voltage should be a function of density × radius for changes in density, if the radius is the determining factor. This law does approximately hold over a moderate range. Edmunds [63] has verified it for points of radius 0·25, 0·05, 0·075 cm.

(*f*) Spark Lag

(i) *Impulse Ratio*

It is found with most forms of electrodes, but particularly with points, that there is a certain time interval between the attainment of the voltage and the passage of the spark. This time interval is known as the *spark lag*. It decreases as the voltage increases above the minimum for breakdown, and also depends upon many other factors.

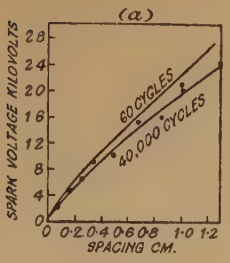
When A.C. is used this lag gives a higher voltage than for D.C., for the time during which the voltage is sufficiently high is limited by the frequency. A measure of the lag can thus be defined as the ratio $\frac{\text{spark potential with given A.C.}}{\text{spark potential with D.C.}}$ This is known as the *impulse ratio*. The impulse ratio must be given for a particular wave form, since the time of duration of high voltage is important: thus the impulse ratio is higher for "peak" forms of voltage than for sinusoidal forms. The matter is further complicated by over-voltage due to oscillations in the circuit.

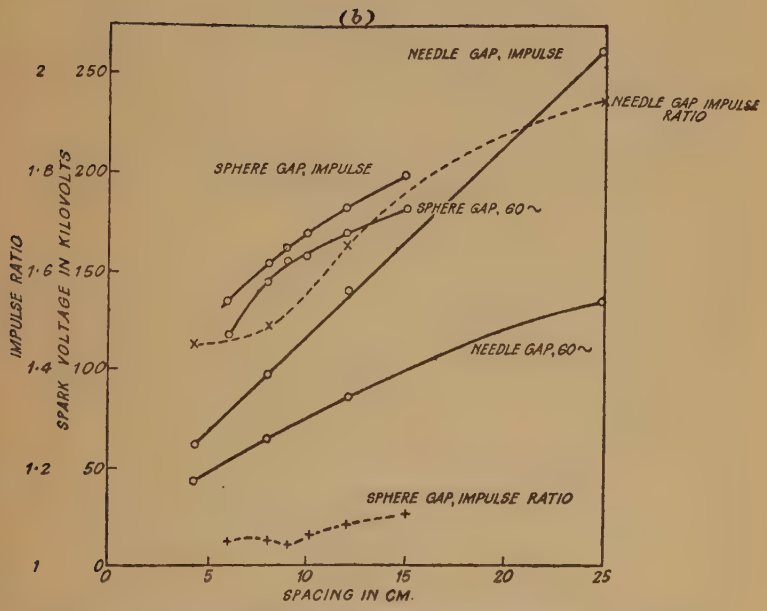
For uniform fields (Leontiewa's [36] result has already been mentioned) the impulse ratio is unity. For spheres, the effect of lag is often masked by over-voltage, as in Diagram XXV (*a*). In Diagram XXV (*b*) curves are given showing the variation of impulse ratio and spark potential with spacing for spheres and needles, taken from Peek.[64] These phenomena will be developed more fully under the different theories with which they have been connected.

[63] Edmunds, *Phil. Mag.*, **28** (1914).

[64] Peek, *Dielectric Phenomena*, p. 87.

DIAGRAM XXV





Sparking Voltages for Sphere Gap and Needle Gap for 60~ and Impulse Voltage. Impulse Ratios calculated for 60~.

(ii) *Casual Ion Theory*

The general view is that spark lag is due to lack of casual ions distributed favourably to form ions by collision, and so initiate the discharge. We can rule out any possibility of the removal of ions from the surfaces of the electrodes, for Almy, Broxon (see (*h*)), and others have shown that to do this requires very high field strengths indeed.

The casual ion theory was first suggested by Edmunds,[65] and Zeleny [66] has elaborated it. According to the latter, we have some kind of transition layer surrounding the point. This layer (water, gas, oxide, etc., *vide infra*) must first be ruptured before the discharge begins. There is a certain "critical" volume, such that if an ion comes within it the field will be sufficiently strong to accelerate it and cause it to penetrate the transition layer, so starting the spark. The higher the voltage the larger the volume and the greater the chance of casual ions coming within it. The lag is therefore less for higher voltages, and less the longer the time the voltage persists.

Zeleny [61, 66] finds that, for needle points prepared in such a way as to render them "slow" or "sensitive," *i.e.* having a large lag, the discharge starts impulsively. The voltage necessary for sparking may vary, but it is never less than a certain minimum, B. If the voltage is now reduced, the discharge having started at a certain voltage C, the "stopping voltage," the current ceases. In between B and C is another voltage A, such that below A no discharge will take place, while between A and B a momentary discharge occurs if the gap is subjected to ionising radiation. Above B normal discharges occur, but with a time lag. "Sensitive" points employed at voltages between A and B are of great value in detecting α particles and similar ionising agents.

With larger points (Gorton and Warburg,[58] Zeleny [57]) it is found, as was to be expected, that if spark lag is due to lack of casual ions, the ionising radiations will make the starting and stopping voltages equal.

[65] Edmunds, *Phil. Mag.*, **28** (1914).
[66] Zeleny, *Phys. Rev.*, **19** (1922).

In his experiments on liquid points, Zeleny [61, 66] has found that the spark lag depends on the pressure and radius of the "point," or drop, according to the following equation :—

$$f=0{\cdot}955p+5{\cdot}60\sqrt{\frac{p}{r}} \quad . \quad . \quad . \quad b8.1$$

where f=retardation of current, p=pressure, r=radius. The effect has been observed with water, glycerine, and alcohol.

(iii) *Pilot Spark*

A commentary on the importance of initial ionisation is found in the practical device of the "pilot spark," [67] or three-point gap, which has been utilised in the construction of a form of standard spark gap. If three points, two of them close together, are arranged so that a discharge passes between the weaker before the stronger gap breaks down, it is found that the presence of the subsidiary gap reduces and, according to some, eliminates the lag of the main gap. On the casual ion theory this is simply explained as due to the supply of initial ions by the pilot spark. It has also been suggested, in opposition to this, that the subsidiary gap induced oscillations and over-voltage in the main gap. Apart, however, from the doubt as to whether an impulse over-voltage would be adequate to cause breakdown, a recent paper [139] seems to have proved that the effect is due to ionisation of the gas in the main gap. Summarising the results of these experiments as a whole, it may be stated that—

1. Before the three-point phenomenon can be obtained, some form of discharge, silent or spark, must be visible from the main gap and within a certain distance of it.

2. The phenomenon is obtained in the absence of (*a*) disturbances of the electric field near the main gap, and (*b*) the passage of ions from the pilot discharge into the main gap.

3. A radiation is emitted by the pilot discharge, which can ionise the gas in the main gap.

4. When this radiation is prevented from reaching the main gap, the usual three-point effect is not obtained.

[67] Campbell, *Phil. Mag.*, **41** (1921).
[139] C. E. Wynn-Williams, *Phil. Mag.* (Feb. 1926).

5. The three-point effect is still produced when the electrodes are so shielded as to prevent any photo-electric effects being shielded by the radiation.

6. The observed properties of this radiation seem to establish the limits of its wave-length as being approximately 1000 Å. and 13 Å., and they coincide to a large extent with the corresponding properties of *entladungstrahlen.*

It is therefore *inferred* that the three-point effect is caused by the ionisation of the gas in the main gap by a radiation, believed to be a form of *entladungstrahlen,* emitted by the pilot discharge. Disturbances of the field, passage of ions into the gap, or photo-electric effects produced by the radiation or by ultra-violet light, while possibly assisting in, are not essential for, the production of the three-point effect. (It is interesting to note that when the radiation is prevented from reaching the gap by means of a quartz plate, photo-electric effects at the electrodes, due to ultra-violet light emitted from an intense pilot discharge and transmitted by the plate, may give rise to a diminished form of three-point effect. Under these conditions, of course, the ultra-violet light is the main cause of the effect.)

If the radiation proves to be a form of *entladungstrahlen,* two interesting facts are disclosed regarding it: (1) that *entladungstrahlen* are produced in air at ordinary pressure by a silent discharge as well as by a spark; (2) that a critically adjusted spark gap subjected to impulsive voltages forms a convenient indicator of the presence of *entladungstrahlen* (when no other indications are present which ionise gases or produce photo-electric effects), and that it appears to be more sensitive in use than either an ionisation chamber or a thermo-luminescent salt.

Incidentally, it is suggested that possibly either or both of the modified types of three-point gap used in this work might, on investigation, prove to be more suitable as a standard type of gap than the usual type employed, in which an insulated pointed third electrode is used. It was noted in the Introduction that discharges often emitted rays of the type of Wiedermann's *entladungstrahlen.*

(iv) *Transition Layers and Nature of the Surface*

That spark lag is not due to the absence or presence of surface irregularities is shown by Zeleny's experiments on liquid points. These, although they must be perfectly smooth, show spark lag just as needle-points do.

Campbell [67] finds that "hardness" or "slowness" of gaps may be produced by rubbing with emery paper. A strong spark, and also preliminary boiling in nitric acid, softens the gap, *i.e.* reduces the spark lag. This latter observation is explicable on Zeleny's theory as removing the transition layer.

According to Zeleny, Meyer,[71] and Stucklen,[69] water-vapour shows a strong tendency to form a transition layer and skins over the electrodes.

Reiche [33] thinks that the current effects near the sparking potential already mentioned (II (*e*) (i)) are due to gas-transition layers forming a condensed layer of electricity on the electrode surfaces. These layers are ruptured in the presence of radio-active material, which gives rise to weak initial currents. Stucklen [69] also favours the presence of gas-layer effects.

Gæde [70] has found polarisation round a point after discharge, as might be expected if there is a surface layer acting in the manner supposed by Zeleny.

(v) *Energy Storage Theory*

Peek has suggested that, in the case of impulse sparking, a certain time is required for the air to store up the energy necessary to carry on the discharge. This storage may be interpreted either as one of kinetic energy in the neighbourhood of the electrodes, or as representing the time for the initial discharge to spread over the electrodes. According to the latter idea, large spheres and plates should show a lag, owing to the large area of the electrodes, but a simple calculation shows that the order of the largest lags possible, according to observations, is much too

[71] Meyer, *Ann. der Physik*, **68** (1921).
[69] Stucklen, *Ann. der Physik*, **65** (1921).
[70] Gaede, *Ann. der Physik*, **4**, 14 (1904).

small, unless the discharge starts simultaneously from several points. In any case, spheres and plates show no appreciable lag.

The first interpretation is rather vague, except that it might be explained as the time necessary to bring the initial corona up to sparking. Campbell [67] has adopted this view, and it will be discussed with his theory.

It is surprising that Peek, throughout his work, is inclined to minimise the effect of ionising radiations on sparking. As has already been mentioned, practically all the other workers disagree with him. In the case of spark lag the evidence is particularly strong, and his view that such radiations decrease the lag without raising the voltage is tenable, only if a fair measure of casual ionisation is assured sooner or later. In some cases sparking will not take place at all, unless an ionising ray or α particle enters the ionisation chamber.

(vi) *Campbell's Theory*

Campbell [67] has adopted Peek's storage theory as a partial explanation. According to him, spark lag should be divided into two parts :

(1) Due to lack of casual ions.
(2) Due to Peek's storage interval.

He goes on to say that the second part is taken up in transforming the initial corona into a spark ; although he says only a partial corona and not a full one need be developed. Wolcott's investigation of the importance of initial glows and brushes, between a point and plane, on sparking seems to support the theory of initial corona, but it does not seem quite clear that the distinction of this corona effect from an arrangement which supplies initial ionisation is important. The distinction between (1) and (2) is, on the whole, not of the strongest.

Campbell finds that if one measures the sparking voltage as the ratio $\frac{Q}{C}$, where Q=quantity of electricity passing in each spark and C=capacity of sparking circuit, it turns out to be the D.C. spark potential. This we may compare with the fact that when a spark passes between electrodes charged by an influence machine

(say) there is a voltage drop on passage of the spark. Accordingly, Campbell's method probably measures this later and smaller voltage, which is independent of initial ionisation and other similar causes of spark lag, etc. This method of measuring spark potentials seems a valuable one.

In those cases where corona can be developed there should be a constant lag for the particular arrangement, for the time to transform corona into spark should be fairly constant. There does not seem sufficient evidence to prove this. Campbell would explain it as due to lack of initial ionisation; but at high pressures the impulse ratio becomes practically unity. If, now, this is explained as due to the fact that the corona and spark potentials are very nearly equal at high pressures, there is still the effect of initial ionisation to come into play. On Zeleny's theory, the voltages being high at high pressures, the critical volume in the neighbourhood of the electrodes is large, and the surface layers rupture more easily.

(vii) *Morgan's Theory*

Morgan [68] thinks it more appropriate to consider one lag depending upon (1) the rate of rise of voltage, (2) the time it must act, (3) the initial ionisation, instead of Campbell's two lags. He employed an induction coil to give the voltage, and, by varying the primary current, was able to show the effect of change of voltage, and consequent steepness of wave, upon the spark lag. If the voltage rises sharply, the gap may spark over at a higher voltage although the lag is reduced, *i.e.* the impulse ratio would be higher although the lag would be less. This is illustrated in Diagram XXVI (*a*). The straight line parallel to the axis of time represents V_0, the D.C. minimum sparking voltage. The curves represent different voltage forms for different primary currents. They are sinusoidal, with the same frequency, but different maximum voltages. The spark lag for primary current $i_0=1$ is AB_1. i_0 is the least primary current for which sparking will take place at all at the given frequency, so one presumes that it occurs at the maximum voltage V_1. Suppose a straight line to be drawn cutting the curves in V_1, V_2, . . . V_6, and suppose

[68] Morgan, *Phil. Mag.*, **41** (1921).

this to be the law for the spark voltage. Then the lags for primary currents (proportional to the maximum voltage) are given by the horizontal projections of V_2R_2, etc. These lags are less than the lag AB_1; in fact, the *lag decreases with the voltage, but the actual voltages are higher*, i.e. *the impulse ratio increases with the steepness of the voltage impulse, although the lag decreases with the rise of the peak voltage.*

If the above construction is assumed the theoretical one XXVI (*b*) gives the relation between the actual sparking voltage and the primary current (*i.e.* the maximum voltage). XXVI (*c*) shows a few experimental curves which bear some resemblance to the theoretical curve.

The effect of steepness is certainly important, both theoretically and practically, and should be investigated, but otherwise Morgan's theory is phenomenological and not fundamental.

Stucklen [69] thinks that the lag should be divided into two parts, one depending on the gas and the other on the nature of the electrodes; but from the experimental evidence it does not seem that this distinction would clarify results to any great extent.

(viii) *Spark Lag and Time Effect*

Spark lag is comparable with the time effect in a solid dielectric breakdown, although arising, in all probability, from quite different causes. It is a phenomenon which may be of great importance in connection with sparking plugs, lightning arresters, safety gaps, etc. In future work it appears desirable to obtain surfaces of determinate form and nature, to observe the surface layers, if any, to apply voltages of accurately known wave-form and strength, and to determine the instant of breakdown directly, if possible. A large number of readings should be taken to detect probability variations, such as Zeleny's theory would lead one to expect.

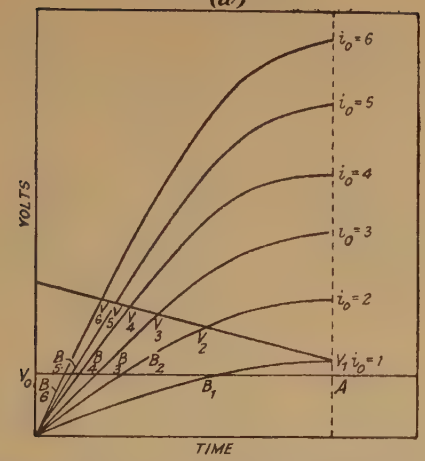
(*g*) Effect of Humidity

(i) *Uniform Fields*

Except as far as increase in density is concerned, humidity appears to have a comparatively small effect upon sparking in uniform fields.

DIAGRAM XXVI

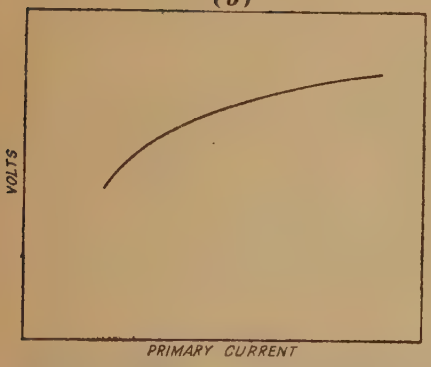
(a)



Effect of Rate of Rise of Voltage on Impulse Sparking.

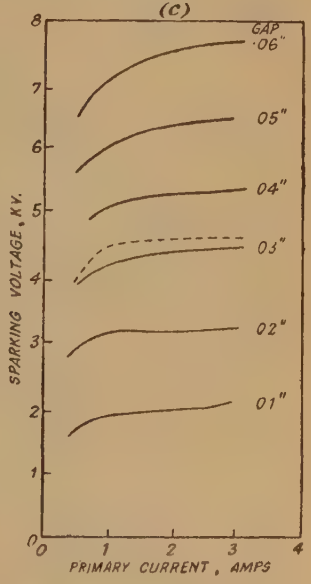
i_0=Primary Current Induction Coil.
i_0=1=least Primary Current for Sparking.
V_1, V_2 to V_6=Voltages at which Sparking occurs.
V_0=Minimum D.C. Spark Voltage.

(b)



Sparking Voltages, *v.s.* Primary Current calculated from (*a*).

(c)



Observed Sparking Voltages for different Primary Currents.

(ii) *Sphere Gap*

The spark voltage of the sphere gap is reduced by rain, or other water, on the surface, as is seen from Diagram XXVII (*a*). Humidity alone appears to have very little effect.

(iii) *Parallel Wires*

Water and oil reduce the voltage as for spheres. This is shown in Diagram XXVII (*b*). The effect of humidity alone is variable, but for fine wires and large spacings it decreases the voltage.

(iv) *Points*

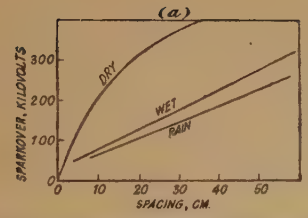
On Diagram XXIV are curves, due to Peek,[64] showing the effect of humidity on the needle gap. The sparking voltages are increased. On Diagram XXVII (*c*) and (*d*) are shown the variations of sparkover voltage for a given gap with humidity, and the sensitivity to humidity at various spark voltages, *i.e.* the change in spark voltage produced by a given percentage change in humidity. It is seen that the needle gap is not very reliable, owing to humidity effects above 30 *k*V.

(v) *Spark Lag*

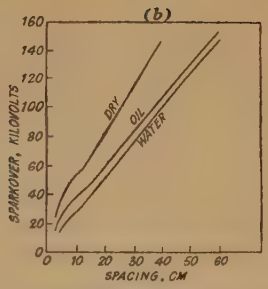
We have seen that, in the opinion of Zeleny,[66] Stucklen,[69] and Meyer,[71] water-vapour has a strong effect on the electrodes, and so increases the spark lag, and at the same time the normal spark potential. It is well known that water-vapour is very efficient in forming surface layers. The difficulty of its removal is a common experience in high vacuum work, and in all work on surfaces, so that it is possible that the effect of humidity lies here.

Another theory put forward is that humidity condenses round the ions and so clogs their action, decreasing their mobilities and ionising powers. Experiments on corona pressures show that, in corona, the proportion of large, heavy ions may amount to as much as 13 per cent., so that it is quite probable that water-vapour acts partly in this manner. However, with high voltages and fields the ions would soon lose their condensed molecules, and appear as electrons or positive ions. Accordingly, one would

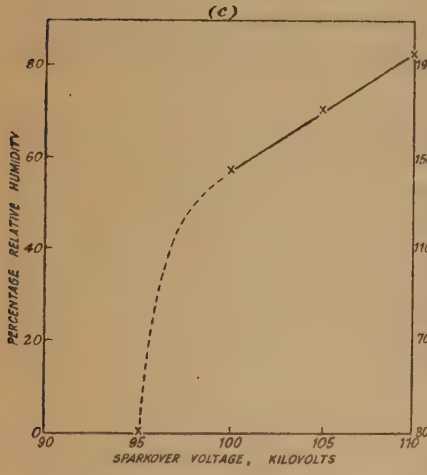
DIAGRAM XXVII



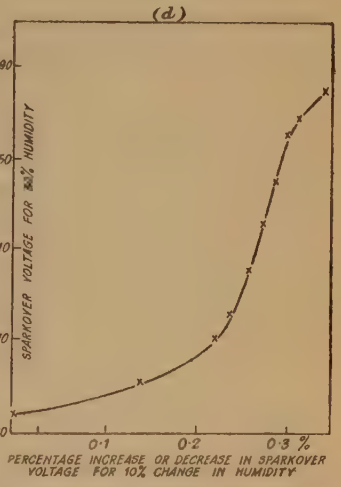
Effect of Water and Rain on Sparkover between 25 cm. Spheres.



Effect of Water and Oil on Surface.



Variation of Sparkover Voltage with Humidity for 25 cm. Needle Gap.



Variation of Sensitivity of Needle Gap, to Humidity Changes, with Voltage.

expect that, for high voltages, water-vapour would tend to decrease in its effect on sparking potential, while its effect on lag remained much the same, the latter being an initial effect.

According to Sægusa,[76] humidity up to 30 per cent. diminishes the sparkover potential for most forms of electrodes. His paper also contains a discussion of the effects of interposed dielectrics, on the view that the dielectrics not only cause flux concentration, but also lower the effective voltages, owing to conduction through them and over their surface.

(*h*) Short Sparks in Air

From the curves for sparking in uniform fields one observes that, for any pressure, there should be a minimum sparking potential for some particular distance. For distances less than this, about 10^{-3} cm. at atmospheric pressure, the spark potential should rise.

Earhert [72] obtained some results apparently at variance with this, finding a spark potential of 300 volts at about 10^{-4} cm.

Almy [73] has shown, however, that in all probability his electrodes touched under the strong electrostatic attraction. He finds that electricity cannot escape from the metal itself (which it would have to do if the spark gap were less than the mean free path) for fields up to 3×10^7 volts/cm.

Hoffman [74] gives the following table for the strengths necessary to remove ions from metals :—

Metal	Pt. Ir. alloy.	Cu. (I).	Cu. (II).	Cu. (III).	Al.	Zn.	Pb.
Field	$4{\cdot}8\times10^6$ volts/cm.	$3{\cdot}8\times10^6$	$3{\cdot}58\times10^6$	$3{\cdot}48\times10^6$	$3{\cdot}4\times10^6$	$2{\cdot}7\times10^6$	$2{\cdot}7\times10^6$

Broxon [75] has shown that no electron atmosphere, such as Wood suggested, can extend beyond $0{\cdot}14\mu$ of the surface, and could obtain no disruptive discharge, at such short distances, up to $1{\cdot}4\times10^5$ V/cm.

[72] *Phil. Mag.* (6), **1** (1901).
[73] Almy, *Phil. Mag.* (6), **11** (1908).
[74] Hoffman, *Zeit. für Physik*, **4**, 3 (1921).
[75] Broxon, *Phys. Rev.*, **70** (1922).

CORONA

I. EXPERIMENTAL LAWS OF CORONA

Corona is usually intended to mean the glow or brush discharge which forms on conductors when the electric stress in their neighbourhood is sufficient to cause local breakdown of the gas. By some writers, however, corona is considered to denote only the brush discharge; others again, notably Peek, use the term to cover all forms of partial breakdown in gases, namely, the glow, brush, and dark discharges.

In the present volume corona is primarily used to connote the glow discharge, but in those cases where the dark discharge or brush forms first, corona means the first discharge which appears. It would perhaps be better that the term corona should be used generally as Peek uses it, but, particularly of recent years, "glows" have been considered to apply to low pressure discharges, and the use of the term might involve, at the present time, some confusion both on this account and because discharges at higher pressures have a much less stable appearance. It is hoped that a standard nomenclature will soon be drawn up.

There is a definite difference between the discharge round positive and negative conductors. In the case of the former an even glow appears, extending to a definite distance from the conductor, which depends on the curvature, stress, and pressure. On negative conductors the glow presents a beaded appearance, the beads being uniformly spaced. Most of the general theory has been worked out by taking the case of the wire and concentric cylinder, but it can be applied, as will be seen, without much alteration to the other cases.

(*a*) Ordinary Pressures

(i) *Wire and Cylinder*

Gaugain [93] early found that the surface stress at which corona formed round a wire inside a concentric cylinder was independent of the length of the arrangement and, provided it was not too small, of the radius of the external cylinder.

The nature of the relation between the corona voltage, or, rather, the surface stress, or force, at the wire surface, and the

[93] Gaugain, *Ann. de Chim. et de Phys.*, 4, 8 (1866).

radius of the wire, has been determined by several experimenters. A.C. corona has been studied with a view to establishing this relation by Peek,[77] Whitehead,[78] Whitehead and Isshiki,[79] Pedersen,[80] and Schumann.[81] D.C. corona has been examined by Watson,[82] Farwell,[83] Whitehead and Brown,[84] Whitehead and Lee,[85] Whitehead and Farwell,[86] and Lee and Kurrelmeyer.[87] On Diagram XXVIII are shown results by Peek,[77] Whitehead,[78] Whitehead and Isshiki,[79] Whitehead and Brown,[84] Whitehead and Farwell,[86] Pedersen,[80] and Schumann,[81] plotted against a logarithmic scale of radius. The agreement is seen to be very good, the results being expressible in three curves, which are nevertheless nearly identical. This may be due to the fact that some investigators used A.C. corona, which is the mean of positive and negative D.C. corona.

However, the results are very closely expressed by a mean curve, and this curve gives for the relation between surface stress and radius the formula for air

$$g_v = 30{\cdot}5\left(1+\frac{0{\cdot}305}{\sqrt{a}}\right) k\text{V/cm.} \quad . \quad . \quad . \quad c1.1$$

where

g_v = stress at wire surface

and

a = radius of wire in cm.

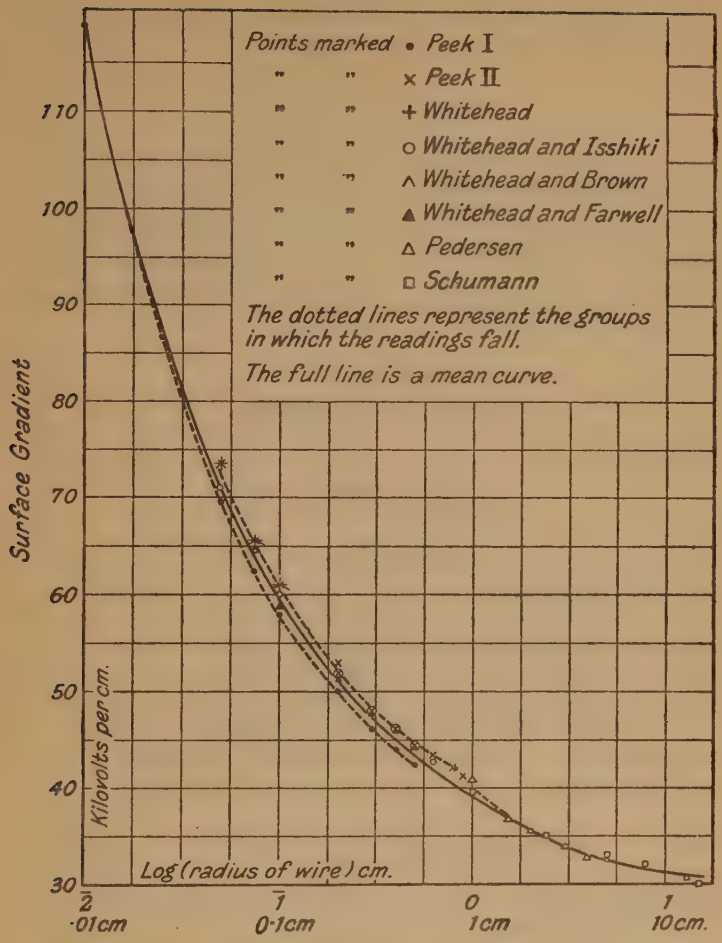
The exact value of the constants is still rather doubtful; *e.g.* Peek gives

$$g_v = 31\left(1+\frac{0{\cdot}308}{\sqrt{a}}\right) k\text{V/cm.} \quad . \quad . \quad . \quad c1.2$$

while others give lower values for both constants: the formula *c*1.1 gives more or less a mean curve.

[77] Peek, *A.I.E.E.* (1910, 1911, 1912); *Dielectric Phenomena*, ch. iii.
[78] Whitehead, *A.I.E.E.* (1910, 1911, 1912).
[79] Whitehead and Isshiki, *A.I.E.E.*, xxxix (1920).
[80] Pedersen, *Ann. der Physik*, **69** (1923).
[81] Schumann, *Archiv für Electrotechnik*, **12** (1924).
[82] Watson, *Electrician* (11th February 1910).
[83] Farwell, *A.I.E.E.*, xxxiii (1914).
[84] Whitehead and Brown, *A.I.E.E.*, xxxvi (1917).
[85] Whitehead and Lee, *A.I.E.E.*, xl (1921).
[86] Whitehead and Farwell, *A.I.E.E.*, xxxviii (1919).
[87] Lee and Kurrelmeyer, *A.I.E.E.* (Jan. 1925).

DIAGRAM XXVIII



Relation between Corona Gradient and Radius at Wire. For Wire in Cylinder (and Concentric Cylinders).

The radius of the external cylinder had no observable effect on the surface corona gradient when the ratio of the radii of cylinder to wire was greater than about 3.

(ii) *Parallel Wires*

The critical gradients for the formation of corona on parallel wires have been examined, as in the previous case. The arrangement is not quite so simple, but, owing to its practical importance, a great deal of work has been done. Peek's [88] formula is probably as accurate as any, as there is not much divergence between results.

He gives

$$g_v = 30\left(1+\frac{0{\cdot}301}{\sqrt{a}}\right) \text{kV/cm.} \quad . \quad . \quad . \quad c1.3$$

where

g_v = stress at wire surface

and

a = radius of wire in cm.

On Diagram XXIX (*a*) is shown g_v plotted against $\frac{1}{\sqrt{a}}$, taken from some of Peek's results to illustrate his law.

Provided it were fairly large compared with the radius of the wires, it was found that the spacing had no influence upon the corona gradient. The exact point at which this ceases to be true is still uncertain.

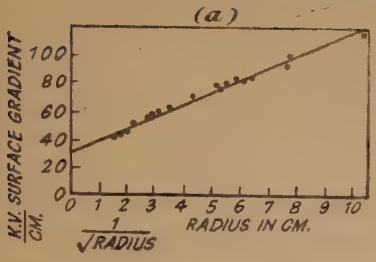
(iii) *Spheres* (see Diagram XXIX (*b*))

Corona forms with difficulty on spheres. According to Peek, it is not visible until the spacing is twice the diameter, and the corona and sparkover voltages do not appreciably differ until the spacing is four times the diameter. However, as was observed in Sparking, III (*b*) (ii), the sparkover gradient at the smaller spacings is continuous with the corona curve. Accordingly, for the range $\frac{\text{spacing}}{\text{radius}} = 0{\cdot}3$ to $2{\cdot}0$, the two experimental laws mentioned hold, namely,

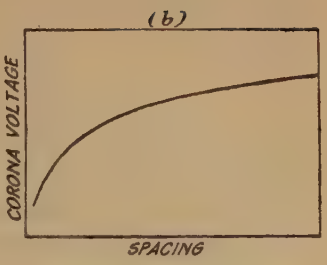
The gradient at the surface is a linear function of the reciprocal

[88] Peek, *Dielectric Phenomena*, p. 48.

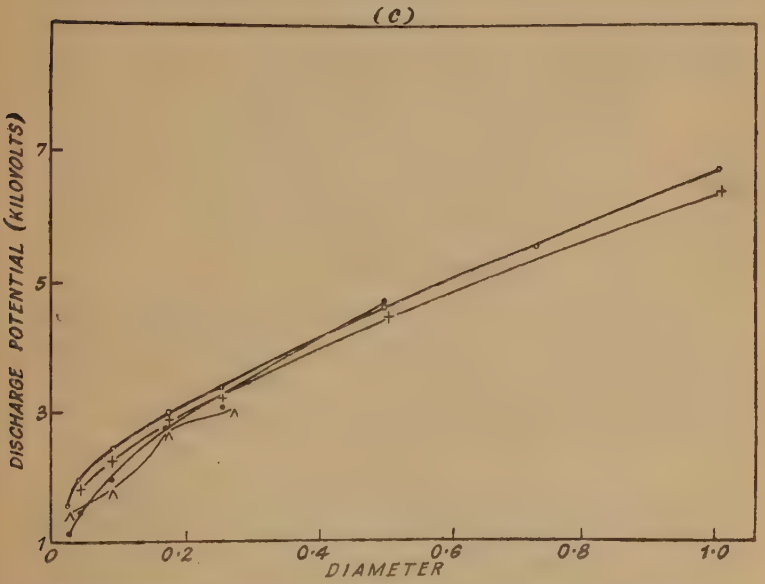
DIAGRAM XXIX



Parallel Wires.



Specimen Curve of Corona Voltage. Spheres.



Relation between Voltage and Diameter for Discharges from Spherical Points (Zeleny).

of the square root of the radius for a given value of the ratio spacing/radius.

Hence

$$g_v = A + \frac{B}{\sqrt{a}} \quad . \quad . \quad . \quad . \quad c1.4$$

and A approaches 30 to 31 *k*V/cm. for values of the ratio spacing/radius approaching 2.

For larger values of the same ratio the corona gradient is very nearly constant. The spheres are here so widely separated, however, that great accuracy cannot be obtained.

Peek states that the surface gradient for discharge is independent of the spacing, for spacings greater than a certain amount. This, although not altogether adequate, is very roughly true. Peek [88] gives the law for spheres as

$$g_v = 27{\cdot}2\left(1 + \frac{0{\cdot}54}{\sqrt{a}}\right) k\text{V/cm.} \quad . \quad . \quad c1.41$$

where

g_v = surface gradient for corona

and

a = radius of wire.

When the spacing becomes very large the arrangement tends to resemble a pair of points, in which case, in the opinion of many, the maximum gradient for corona is independent of the spacing. The experimental evidence is, however, not very definite.

(iv) *Points*

The first discharge to take place from a point is a dark discharge. When the potential is sufficiently raised, this turns into a glow or corona which, however, may or may not be stable. Thus, if the point is too near the opposite electrode, the discharge becomes intermittent, and brushes form. At greater distances stable corona is formed. The distance at which stability is attained is less for positive points than for negative points, and the blunter the point the greater must be the spacing.

No laws have been formulated to represent the phenomena, but on the whole the potentials are less the sharper the point, and are less for points with flat ends than for those with rounded ends, due to the greater flux concentration at the edges. In

Diagram XXIX (*c*) are shown the results obtained by Zeleny [90] for wires of different diameters with hemispherical ends.

(v) *Effect of Pressure*

On the whole, we see that the corona gradient depends only on the curvature of the surface at which corona forms. Accordingly, the effect of pressure may be conveniently summarised as causing a change in density which affects the discharge according to the general discharge theorem; *i.e.* the corona potential or the $\frac{\text{corona gradient}}{\text{density}}$ is a function of the density multiplied by the radius of curvature. If the gas obeys Boyle's law, and if the temperature is constant, this may be expressed in the form

$$\frac{\text{corona gradient}}{\text{pressure}} = \text{function (pressure} \times \text{radius}) \quad . \quad c1.5$$

For the wire and cylinder this leads to the formula

$$g_v = 30{\cdot}5\mathrm{P}\left(1+\frac{0{\cdot}305}{\sqrt{\mathrm{P}a}}\right) k\text{V/cm. (see } c1.1) \quad . \quad c1.6$$

where P is the atmospheric pressure.

With Peek's values this becomes

$$g_v = 31\mathrm{P}\left(1+\frac{0{\cdot}308}{\sqrt{\mathrm{P}a}}\right) \quad . \quad . \quad . \quad c1.7$$

This equation has been verified by the following observers, and holds, down to pressures of a few centimetres, as far as the form is concerned, different experimenters disagreeing as to the exact values of the constants: Peek,[91] Watson,[82] Whitehead,[78] Farwell,[83] Brown and Whitehead,[84] Whitehead and Isshiki,[79] Lee and Kurrelmeyer.[87] Some of the results for D.C. and A.C. corona in air and for D.C. corona in other gases are given in Diagram XXX.

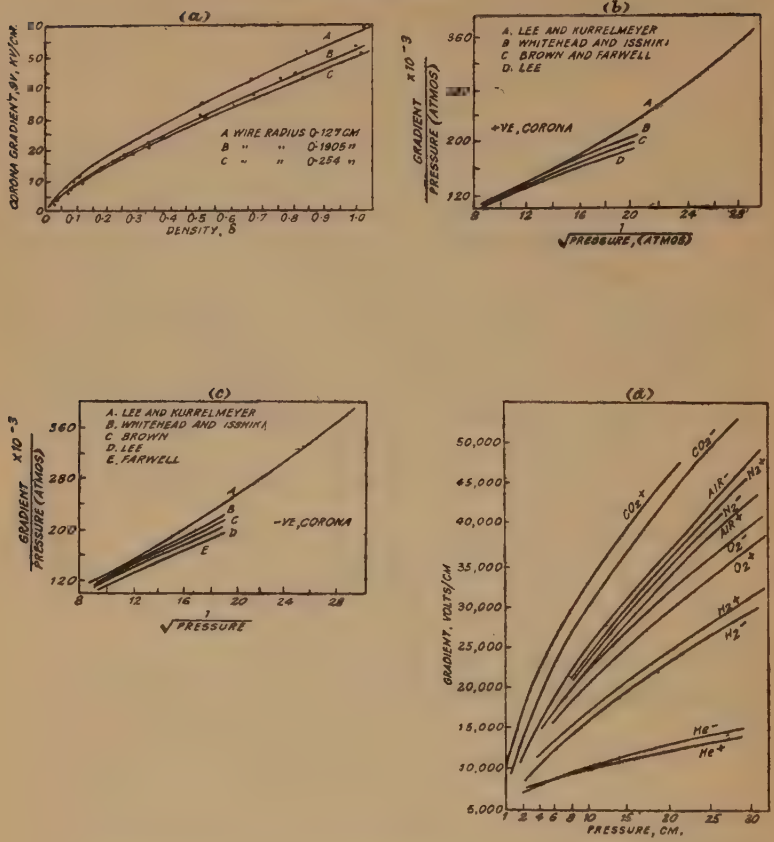
For parallel wires Peek's [91] formula becomes

$$g_v = 30\mathrm{P}\left(1+\frac{0{\cdot}301}{\sqrt{\mathrm{P}a}}\right) \quad . \quad . \quad . \quad c1.8$$

and has been verified, as for wire and cylinder, down to pressures

[91] Peek, *Dielectric Phenomena*, p. 57.

DIAGRAM XXX



Relation between Corona Gradient and Pressure of Air for Wires.

of 10 or 15 cm., by Peek and others. This arrangement is inconvenient for tests at lower pressures.

In the same way, for spheres, we shall have the formula

$$g_v = 27{\cdot}2\mathrm{P}\left(1+\frac{0{\cdot}54}{\sqrt{\mathrm{P}a}}\right) \quad . \quad . \quad . \quad c1{\cdot}9$$

The above holds fairly well over a limited range, but, owing to the unbalanced nature of the field, this formula, as are other formulæ for spheres, is not so satisfactory as for the better balanced fields of parallel wires. (A more complete discussion of this is given in Sparking, III (*b*) (v).)

If corona around points depends only on their radius, then by the general discharge theorem, as above, the discharge potential should be a function of density × radius. This has been verified by Edmunds,[65] for the rounded ends of wires of different diameters, to an accuracy of about 3 per cent.

(vi) *Effect of Temperature*

As was said about discharges in general in the Introduction, temperature has no effect except in so far as it alters the density of the gas.

If δ=density of air, p=pressure in cm., and t=temperature in °C.,

$$\delta = \frac{3{\cdot}92p}{273+t} \quad . \quad . \quad . \quad . \quad c1{\cdot}10$$

with similar formulæ for other gases.

Thus we may represent the effect both of temperature and pressure by the formula

$$\frac{\text{corona gradient}}{\frac{\mathrm{A}p}{273+t}} = \text{function}\left(\frac{\mathrm{A}p}{273+t} \times \text{radius of curvature}\right) \quad c1{\cdot}11$$

where A=constant for the gas (3·92 in case of air).

For wire and cylinder in air this gives

$$g_v = \frac{3{\cdot}92h}{273+t}30{\cdot}5\left(1+\frac{0{\cdot}308}{\sqrt{a\frac{3{\cdot}92p}{273+t}}}\right) \quad . \quad . \quad c1{\cdot}12$$

with similar formulæ for other arrangements.

[65] Edmunds, *Phil. Mag.*, **28** (1914).

Peek [92] and Whitehead and Isshiki [79] have verified the effect for A.C. corona, while Whitehead and Lee [85] have verified it for D.C. corona. Some of Peek's results are given on Diagram XXXI.

(vii) *Note on A.C. Corona*

A.C. corona is the mean of positive and negative D.C. corona, but the component with the lower starting voltage will have a preponderating effect. As the pressure decreases, the curves for positive and negative D.C. corona cross, and here is found (Whitehead and Isshiki [79]) a break in the A.C. curve, due to the shift of preponderating influence from positive to negative corona. This point will be further developed in Section III.

(viii) *Field in Corona*

Booth [94] has explored, by means of an insulated wire, the distribution of potential between a wire and a cylinder while corona is taking place. Some of his results are represented on Diagrams XXXII and XXXIII. All the curves have certain main features in common:

(1) There is a sharp fall of potential at the wire, *i.e.* the field near the wire is intense, and the higher the voltage the further out does this intense field extend.

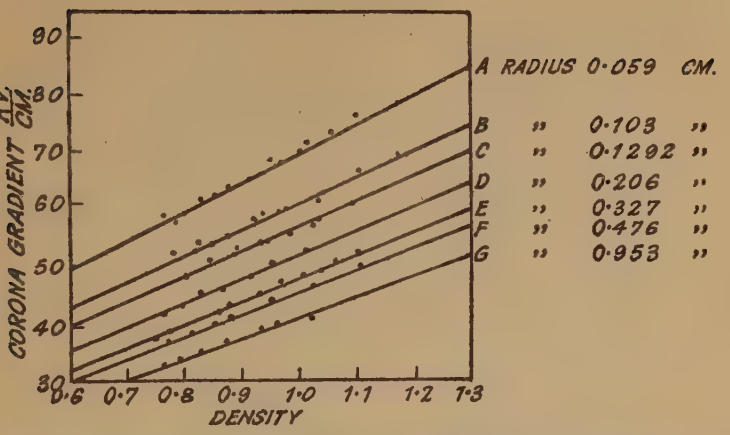
(2) Surrounding the intense field is a large volume within which the voltage fall is such that the field is nearly constant. The higher the voltage the further this portion extends, and the better marked and constant is the field.

(3) Near the cylinder there appears a fairly small area where the voltage is constant, *i.e.* the force is zero. In fig. 1, Diagram XXXII, the curve for barely perceptible glow shows that, when corona first forms regions, (2) and (3) practically merge into a smooth and continuous voltage curve.

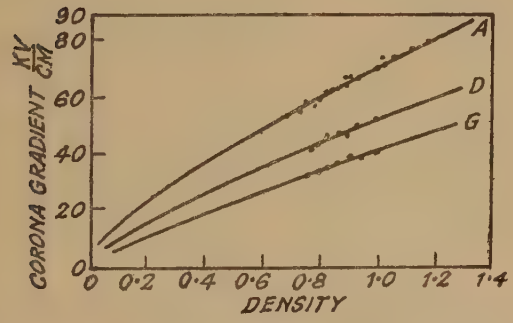
(4) At the actual surface of the cylinder is a sudden change of potential, similar to that at the wire, and consequently there is a strong field on the cylinder surface. It does not extend far, however, and is not of the same order as that on the wire. As the

[92] Peek, *Dielectric Phenomena*, p. 51.
[94] Booth, *Phys. Rev.*, **10** (1917).

DIAGRAM XXXI



Effect of Temperature on Corona Gradient. The effect of Temperature is represented by the consequent Change in Density. Curves calculated according to Peek's Law. $\frac{\text{Gradient}}{\text{Density}} = 31\left(1 + \frac{0{\cdot}308}{\sqrt{\text{radius} \times \text{density}}}\right)$. Points measured.



Effect of Temperature on Corona Gradient. The effect of Temperature is represented by the consequent Change in Density. Curves calculated according to Peek's Law (above). Points measured.

DIAGRAM XXXII

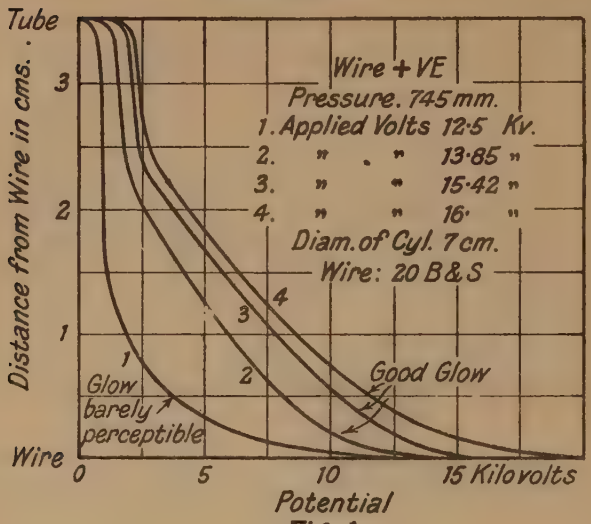


Fig. 1.

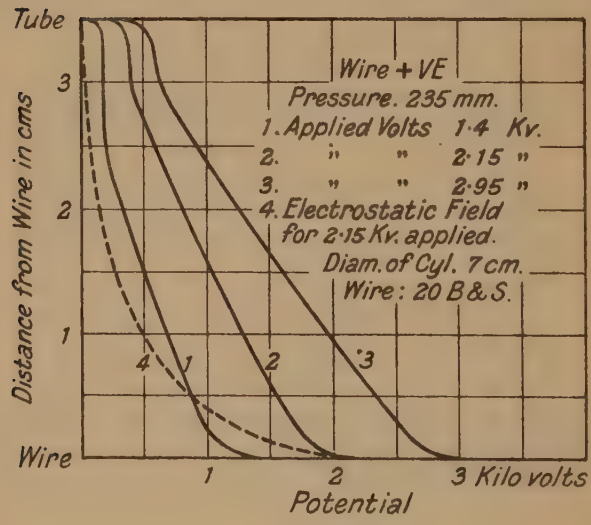


Fig. 2.

Potential Distribution in Corona Potential taken up by an Insulated Wire placed in Field between Wire and Cylinder.

DIAGRAM XXXIII

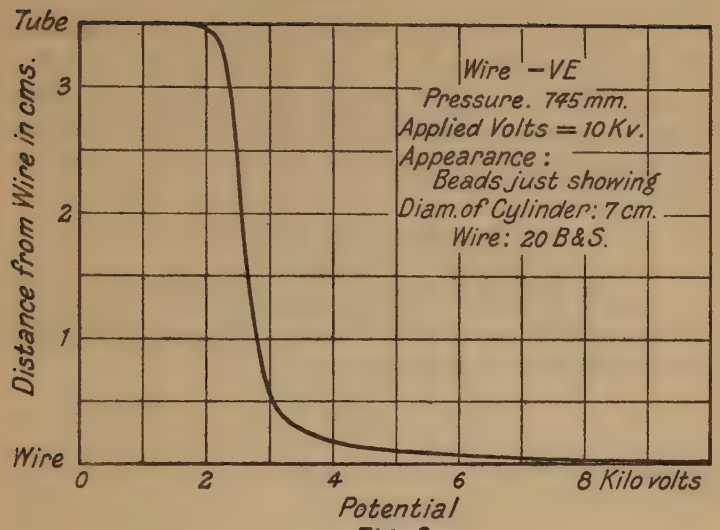


Fig. 3.

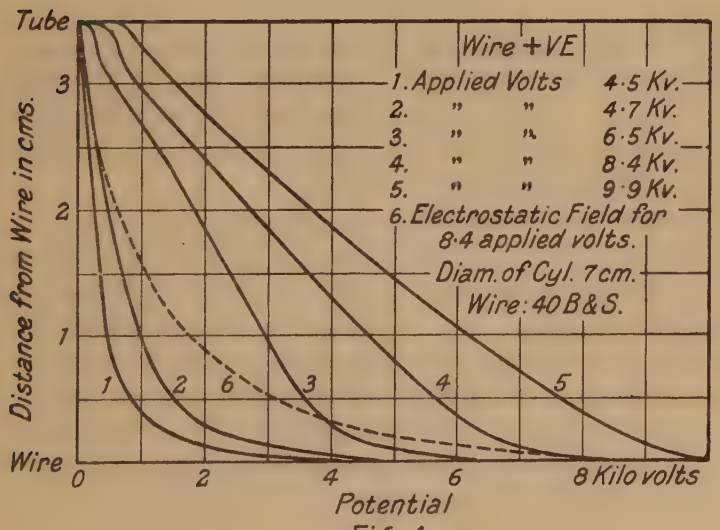


Fig. 4.

Potential Distribution in Corona Potential taken up by an Insulated Wire placed in Field between Wire and Cylinder.

voltage increases this voltage drop at the cylinder also increases. No definite explanation of this is forthcoming.

The curves (except fig. 3) are all given for positive corona, as, owing to the beads, it was difficult to obtain satisfactory results with negative corona. Fig. 3 shows that for negative corona, the configuration is probably fairly similar.

(*b*) Low Pressures

(i) *Lee and Kurrelmeyer's* [87] *Experiments*

When pressures become of the order of a few centimetres, deviations are observed from the law

$$\frac{g_v}{P} = A + \frac{B}{\sqrt{P}},$$

where

P is the pressure

and

g_v the surface gradient,

and for lower pressures the law can no longer be said to hold. On Diagram XXXII are shown some curves by Lee and Kurrelmeyer for a fair range of pressures, and it is seen that for the lower pressures the surface gradient for corona becomes relatively too high to obey the law.

Further, in the cases of those gases, such as air and oxygen, where the negative corona occurs at higher voltages than the positive corona, at a certain pressure the curves for positive and negative discharges cross, the negative corona occurring subsequently more easily. For those gases, however, where negative corona occurs more easily at ordinary pressures, the curves do not cross, *e.g.* hydrogen, nitrogen, carbonic acid. Whitehead and Isshiki's [79] observation of the break in A.C. corona, where the positive and negative curves cross, has been mentioned.

(ii) *Meservey's* [95] *Experiments*

At still lower pressures, of the order of a millimetre or a fraction of a millimetre, characteristic effects are obtained. Meservey has plotted starting voltages against pressure (Diagram XXXII),

[95] Meservey, *Phil. Mag.* (6), **21** (1911).

obtaining curves which show well-marked minima. There is a considerable difference between positive and negative corona. The voltage curve for positive corona is, on the whole, higher than for negative corona, but for fine wires the curves cross, while for thicker wires the curve for negative corona completely surrounds the positive corona curve. Thus for fine wires there appear to be two crossings, one in the neighbourhood of a centimetre or so, as found by Lee and Kurrelmeyer, and one at a fraction of a millimetre, according to Meservey. It is a pity that Lee and Kurrelmeyer did not extend their measurements to lower pressures in order to test this.

MacKenzie [96] has also some interesting comments. He found Peek's equation verified down to moderately low pressures. At lower pressures there is a considerable difference between positive and negative corona. While positive corona is formed at all pressures, negative corona for thick wires may not be formed at all, but only a vacuum discharge may occur. On raising the voltage in positive corona it may, if the pressure be low enough, be transformed into a vacuum discharge. Crooker considers in this connection that the beads in negative corona are analogous to the cathode spot in the vacuum discharge.

II. THEORIES OF CORONA

There are two main theories, Peek's and Townsend's, which have been fairly fully developed. They lead, in many cases, to identical equations, so that some of their applications appear merely repetitions.

(*a*) Peek's Theory [87]

(i) *Law of Corona*

According to Peek, air and other gases have a constant "breakdown strength," but discharging does not necessarily ensue when the electric stress exceeds this value. The stress must exceed the breakdown strength for a certain distance from the electrode, depending on the curvature of the latter. This distance is known as the "energy distance" or "storage length," and here the energy is gained to carry on the discharge. It also

[96] MacKenzie, *Phys. Rev.*, **8** (1915).

appears that this energy distance determines the thickness of the film of ionisation surrounding the conductor (photographic results seem to indicate this), but the main point in Peek's theory is that its existence permits the development of sufficient ionic energy (see Appendix VIII). For the "balanced" field of a wire inside a cylinder, the energy distance c is given by $0{\cdot}3\sqrt{a}$, where a is the radius of the wire. Relations of the same form, but with slightly different numerical values of the constant, hold for other fields.

As the air where ionisation is taking place is conducting, Peek looks on it as an extension of the conductor, the whole being the "equivalent coronal conductor."

We observe that these considerations imply that only the curvature of the corona-forming electrode determines the corona-gradient.

(ii) *Wire in Cylinder and Concentric Cylinders*

If g_0=minimum breakdown strength, g_v=gradient at wire surface, a=radius of wire, A=radius of cylinder, V=applied voltage, then

$$g_v=\frac{\mathrm{V}}{a\log\frac{\mathrm{A}}{a}}.$$

$$g_0=\frac{\mathrm{V}}{(a+c)\log\frac{\mathrm{A}}{a}}.$$

For a distance c from the wire the gradient falls to g_0.

$$\therefore\quad g_v=g_0\left(1+\frac{c}{a}\right)=g_0\left(1+\frac{0{\cdot}3}{\sqrt{a}}\right)\qquad .\qquad .\qquad c2.1$$

if $c=0{\cdot}3\sqrt{a}$.

Comparing this with Peek's experimental equation $c1.2$, namely,

$$g_v=31\left(1+\frac{0{\cdot}308}{\sqrt{a}}\right),$$

we see that, according to Peek's theory, air has a breakdown

strength of about 31 kV/cm., and that the empirical assumption that the energy distance c is given by

$$c=0{\cdot}3\sqrt{a} \quad . \quad . \quad . \quad . \quad c2.2$$

fits in with experiment.

Lastly, the diameter of the outside cylinder has no effect on the critical surface gradient. This has already been noted as an experimental fact.

(iii) *Parallel Wires*

If

a=radius of the wire,
S=spacing,

where g_v and g_0 have their previous meanings, the field g at a distance x from the surface of a wire in the line of centres is given by

$$g=\frac{V\sqrt{S^2-4a^2}}{\{(a+x)(S-2a)-a^2\}\log\left(\frac{S}{2a}+\sqrt{\frac{S}{4a^2}-1}\right)}.$$

If a and x are small compared with S, this becomes

$$g=\frac{V}{(a+x)\log\frac{S}{a}} \quad . \quad . \quad . \quad c2.2$$

This equation is identical with that for wire and cylinder,

$$g=\frac{V}{(a+x)\log\frac{A}{a}},$$

except that the spacing S replaces the diameter A of the cylinder.

Accordingly, for relatively large spacings, the law of corona should be the same as for the wire and cylinder as far as the surface gradient is concerned. For the surface gradient is independent of the external diameter, which is the only point in which the two field equations differ. Experimentally, we have seen that for parallel wires

$$g_v=30\left(1+\frac{0{\cdot}301}{\sqrt{a}}\right)k\text{V/cm. (see I } (a) \text{ (ii), formula } c1.3).$$

This is practically identical with the form for wire and cylinder and so supports the theory.

Analogously with the previous case, the surface gradient is independent of the spacing.

(iv) *Spheres*

Peek uses the following formula by Dean for the gradient at a point on the axis distant y from the surface of one or two equal spheres of radius a :—

$$g_v=\frac{V}{S}2S^2\frac{[S^2(f+1)+4\left(\frac{S}{2}-y\right)^2(f-1)]}{[S^2(f+1)-4\left(\frac{S}{2}-y\right)(f-1)]^2} \quad . \quad . \quad c2.3$$

where S=spacing.

He then approximates, making

$$g_v=\frac{V}{a+2y}\left(\frac{a}{S}\right)f \quad . \quad . \quad . \quad c2.31$$

By putting $g_v=g_0$ where $y=0{\cdot}3\sqrt{a}$ (the energy distance), he obtains

$$g_v=g_0\left(1+\frac{0{\cdot}6}{\sqrt{a}}\right) \quad . \quad . \quad . \quad c2.32$$

His experimental equation is

$$g_v=27{\cdot}2\left(1+\frac{0{\cdot}54}{\sqrt{a}}\right)\text{kV/cm.}$$

The approximation *c*2.31 is, however, unjustifiable. Instead, we may take the two cases : where $S-2y$ is small (*i.e.* where the energy zones nearly touch, which occurs when the field is changing from the effectively uniform) (see Sparking, III (*b*) (ii)), and where y is small compared with S (which is true for larger spacings).

We then obtain

$$g_v=\frac{f(f+1)g_0}{2}\left\{4-2f+\frac{12\beta}{\sqrt{a}}\frac{(f-1)}{\frac{S}{a}}-\frac{12\beta^2}{a}\frac{(f-1)}{\frac{S^2}{a^2}}\right\} \quad c2.4$$

where $\beta=\dfrac{\text{energy distance}}{\sqrt{\text{radius}}}$.

If β is small, then for a given $\frac{S}{a}$, g_v is expressible in the form

$\left(\text{since } f \text{ is a function of } \frac{S}{a}\right)$

$$A+\frac{B}{\sqrt{a}},$$

where $A<g_0$, *i.e.* <31. A will increase with $\frac{x}{a}$.

This is in qualitative agreement with experiment.

In the second case, where y is small,

$$g_v=g_0\left\{1+\frac{2y}{a}\frac{\left(2f-1-\frac{1}{S}\right)}{\frac{S}{a}}\right\} \quad . \quad . \quad c.25$$

Putting $y=\beta\sqrt{a}$ for the energy distance,

$$g_v=g_0\left\{1+\frac{2\beta}{\sqrt{a}}\frac{\left(2f-1-\frac{1}{f}\right)}{\frac{S}{a}}\right\}.$$

This also is in qualitative agreement with experiment, for the A in the formula

$$g_v=A+\frac{B}{\sqrt{a}}$$

tends towards 31 for larger spacings. Numerical agreement, however, is not satisfactory, since a value must be given for β much differing from that required in other cases.

(v) *Effect of Pressure and Temperature*

It has already been observed that, if the general discharge theorem is applied to Peek's laws of corona, we obtain the relation with pressure and temperature.

Referring to the form of the general discharge theorem given in *a*3.2, we may include the effect of density by writing the previously obtained equations for wire and cylinder, parallel wires, and spheres respectively,

$$\left.\begin{aligned} g_v &= 31\delta\left(1+\frac{0{\cdot}308}{\sqrt{\delta a}}\right) \\ g_v &= 30\delta\left(1+\frac{0{\cdot}3}{\sqrt{\delta a}}\right) \\ g_v &= 27{\cdot}2\delta\left(1+\frac{0{\cdot}54}{\sqrt{\delta a}}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad c2.6$$

where

g_v=corona gradient,
a =radius,
δ =density.

As we have expressed the results in terms of density, the effect both of pressure and temperature are included, for they alter matters only by reason of the changes of density they produce.

These considerations are not actually confirmations of Peek's theory, for they depend only on the general discharge theorem, and, provided the original formulæ are right at constant density, the variation with density will be expressed by the derived equations. The experimental results are nevertheless an interesting confirmation of the general discharge theorem.

We may, however, draw two deductions from these results as to Peek's theory. In the first place, the breakdown strength g_0 must be proportional to the density, while the energy distance must be inversely proportional to the square root of the density.

(vi) *Low Pressure Effects and Discharges from Points*

The divergences from the laws of corona which appear at low pressure, as described in Section I, are not explicable on Peek's theory without making further assumptions which would impair the generality of his considerations.

It has been observed that no law has been formulated for points, so that these lie outside the range of Peek's theory also.

(vii) *Breakdown Strength*

In the Introduction a physical significance was suggested for the term " breakdown strength," namely, the field under which an ion would require, during its mean free path, sufficient energy

to ionise a molecule with which it collided; but breakdown strength has not this meaning in Peek's theory, for, taking the value 30 *k*V/cm., which is usually agreed, the ionisation potential for air at atmospheric pressure, with a mean free path of about 10^{-5} cm., would be about 0·3 volts—very much too low. In any case, since ionisation by collision would occur long before such a gradient (as defined in terms of ionisation potential and mean free path) was reached, owing to some ions having longer paths than the mean, we cannot connect breakdown strength in a simple manner with the characteristic ionisation potential.

Can it be defined as the field necessary to produce breakdown in uniform fields? A glance at the curves for uniform fields shows that the critical gradient is always larger than 30 *k*V/cm., and varies with the distance, reaching a minimum and then increasing. The minimum is itself greater than 30 *k*V/cm.

Accordingly, we see that the term breakdown strength cannot be immediately connected with any of the characteristic physical properties of a gas.

(viii) *Energy Storage*

To account for the fact that, in actual practice, a gas never does break down when the maximum stress just reaches the breakdown stress, Peek introduces the conception of energy storage. The stress must exceed the breakdown strength over a certain distance, so that, within the volume so defined, sufficient energy may be acquired for the discharge to continue. By making this "energy distance" an arbitrary function of the radius, Peek is able to account for the main phenomena; but this function remains empirical, and further, we cannot have the same function at lower pressures, for there we see divergences arise.

With parallel planes the energy distance should become infinite with the radius of curvature. Peek explains this anomaly by pointing out that, in this case, corona does not form. It seems, however, more reasonable to state that here the conception of breakdown strength and energy distance collapse together into the voltage, which, after all, is here the determining factor, and can be considered as a measure of the energy involved.

In any case, the conception of energy storage has not a great deal of meaning. If it is defined as that energy necessary to

ionise a certain number of molecules, it seems unnecessary to talk of energy storage and energy distance when the phenomena become much more real and definite if we use, instead, the number and distribution of the ions.

(ix) *Practical Importance of Breakdown Strength*

The importance of the quantity breakdown strength (30 *k*V/cm. for air) lies in the fact that it does appear to decide where ionisation by collision shall cease, in the case of partial discharges such as glows. For the spark, owing to its essential instability, we cannot say anything very definite about the gradient conditions; but when ionisation by collision takes place only over a limited volume, these limits seem to be determined by whether the electric stress exceeds the breakdown strength or not. One must, however, always remember to consider the discharge as a whole in order to find its complete conditions; air may not break down at less than 30 *k*V/cm. at atmospheric pressure, but the whole configuration of the field determines whether discharge will actually take place.

The above consideration shows that we may also look on the breakdown strength as the discharge gradient under limiting conditions. Thus it would give the discharge gradient for infinitely long uniform fields, corona on infinitely thin wires, etc., provided we could extrapolate the equations holding under ordinary circumstances. This we cannot do, as the laws cease to hold as they approach the limit. The reason is that a discharge can never be taken to a limit, as there must be a commensurable quantity of electricity passing. In other words, there must be a certain thickness of gas to be ionised and an appreciable number of ions must be formed. As Whitehead [78] has put it, a cumulative effect of contiguous layers must be present.

(x) *Kunz's* [99] *Formula*

Kunz suggested that Peek's energy storage might be considered as analogous to the electrostatic energy stored in a dielectric under stress.

[99] Kunz, *Phys. Rev.*, **8** (1916).

Let E=energy,
a=radius of curvature of electrode,
g_v=surface gradient,
g_0=breakdown strength,
c=energy distance.

Then, on this theory,

$$E=\frac{(g_v-g_0)^2}{8\pi}k2\pi ca,$$

where k is a constant analogous to the specific inductive capacity of an electrostatic field.

Thus

$$g_v=g_0\left(1+\frac{1}{\sqrt{a}}\sqrt{\frac{4E}{kcg_0^2}}\right) \quad . \quad . \quad . \quad c2.7$$

In order to agree with the experimental laws of corona, either the energy E and the energy distance c must both be independent of the radius a, which, as far as c, the thickness of corona, is concerned, we know to be untrue, or else $\frac{E}{c}$ is independent of a. The latter would be possible only if $\frac{E}{c}$ and a were proportional to the same function of a, which is improbable at the least. In any case, the idea has not been carried any further.

(xi) *Equivalent Coronal Conductor*

Booth's [94] experiments on the field in corona show that there is a large voltage drop and consequent strong field near the wire during corona. Accordingly, the view that corona is an extension of the conductor is rather unsatisfactory, since, although the air is conducting, it presents actually a higher resistance in the corona volume than it does anywhere else. The conception is, of course, a practical one, and its utility is decided by its applicability; but we shall see, in considering effects such as the capacity effect of corona and the relation of corona with the spark, that the idea is likely to be misleading. Accordingly, the term must be used with considerable care.

[94] Booth, *Phys. Rev.*, **10** (1917).

(*b*) Townsend's[100] Theory

(i) *Connection with Sparking*

The main feature of Townsend's theory is that he connects corona with sparking in uniform fields. This seems, *a priori*, a reasonable plan to follow, since one would expect the common features of discharges to appear most simply in uniform fields. In the case of sparking in uniform fields the phenomenon corresponds to a definite theoretical departure, *i.e.* one in which the current calculated in a simple manner from first principles becomes infinite. This means that the assumptions previously made no longer hold. A new set of laws come into operation. These, surely, are in all probability those which govern all discharges. Further, in a uniform field, when we know our previous assumptions have failed, such laws should appear in their simplest form; and thus the experimental results for uniform fields should contain them in the most applicable form.

Townsend makes the specific assumptions: (1) that ionisation by collision occurs from the corona-forming electrode to the place where the stress becomes equal to the minimum breakdown strength; and (2) that between the electrode surface and the surface so defined a spark discharge takes place. If, now, we apply the experimental spark laws to the quantities so determined, we obtain equations practically identical with Peek's.

Townsend's theory can be extended qualitatively to all discharges, *i.e.* they may all be considered as cases of partial or limited sparking. The theory becomes clearer when particular cases are examined.

(ii) *Wire and Cylinder*

Let g_v=gradient at wire surface,
g_0=minimum breakdown strength,
a =radius of wire,
A=radius of cylinder,
c =distance from wire where the gradient is g_0,
V=applied voltage.

---

[100] Townsend, *Electricity in Gases*, pp. 368 *et seq.*

$$g_v = \frac{V}{a \log \frac{A}{a}},$$

and

$$g_0 = \frac{V}{(a+c) \log \frac{A}{a}}.$$

Therefore

$$\frac{g_v - g_0}{g_0} = \frac{c}{a}.$$

We may write the sparking equation for uniform fields over a certain range as

$$g = g_0 + \frac{1{\cdot}35}{S} \quad . \quad . \quad . \quad . \quad c3.1$$

where g is the sparkover gradient for length S, and $g_0 = 30$ kV/cm., *i.e.* the value generally agreed to be the breakdown strength.

If sparking takes place over the distance c the mean gradient is

$$\frac{g_v + g_0}{2} = g_0 + \frac{1{\cdot}35}{S}.$$

Hence, putting $g_0 = 30$,

$$g_v = 30\left(1 + \frac{0{\cdot}3}{\sqrt{a}}\right) \quad . \quad . \quad . \quad c3.2$$

This shows remarkable numerical agreement with the experimental equation *c*1.1, namely,

$$g_v = 30{\cdot}5\left(1 + \frac{0{\cdot}305}{\sqrt{a}}\right).$$

Townsend's theory, as described, is really an application of his continuity theorem (Introduction, III (*c*)) as interpreted in the Introduction.

The integral in the continuity condition

$$\int_0^l \alpha e^{\int_0^x (\beta - \alpha) dx} dx = 1 \text{ (see } a3.3) \quad . \quad c3.0$$

is only finite for those parts of the field where ionisation is taking place. In the case of corona the integral is from the corona-forming electrode to the place where the discharge ceases. *c*3.0 is therefore the condition for a discharge between the electrode as above and an electrode coinciding with the equipotential

surface at the limit of corona and of the same potential. Since in this hypothetical discharge the ionisation would be continuous from electrode to electrode, it would be a spark discharge. Thus the equation *c*3.0 leads to the view that limited sparking takes place from the corona-forming electrode to the limit of the discharge, which Townsend assumes is given by the fact that the field falls below a certain critical value.

Schumann * uses directly his simplified form of *c*3.0, viz.

$$\int_0^l \alpha dx = \text{const.} \quad . \quad . \quad . \quad . \quad c3.01$$

and therefore is following the same lines as Townsend. Townsend's theory, however, has the advantage, as mentioned elsewhere in the present section, that we need only substitute experimental sparkover results without making assumptions as to values of the coefficients of ionisation by collision. On the other hand, the limit of the discharge is fixed rather arbitrarily.

(iii) *Parallel Wires*

When this arrangement was discussed under Peek's theory, it was found that the equations for corona-gradient in this case were identical with those for the wire and cylinder. The only difference in the field equations close to the wire was that the spacing replaced the diameter of the cylinder. The gradient is independent of these latter quantities in each case.

Accordingly, on Townsend's theory, as on any other, the two arrangements (provided the spacing is sufficiently large for the approximations to hold good for parallel wires) should give identical results, and the remarks made on the subject when Peek's theory was discussed apply equally here (see I (*a*) (ii) and II (*a*) (iii)).

(iv) *Spheres*

Let g_v=gradient at surface for discharge,
g_0=breakdown strength as before,
g =critical tension,
V=applied voltage,
a =sphere radius,
S =spacing.

* Schumann, *Durchbruchfeldstarke von Gasen* (Berlin, 1923).

We shall use the approximate formula *c*2.3 for the gradient at any point on the axis, and as in the discussion of Peek's theory, take the two cases: (1) when S−2*c* is small, and (2) when *c* is small. In the former case we obtain

$$g_v = f^{\frac{f+1}{2}} g_0 \left[(4-2f) + \frac{24a}{(g_v-g_0)a} \frac{f-1}{\frac{S}{a}} - \frac{48a^2}{(g_v-g_0)^2a^2} \frac{(f-1)}{\frac{S^2}{a^2}} \right] \quad c3.3$$

a=constant=1·35, approximately, when g_v is expressed in *k*V/cm.

This gives approximately the experimental law that for a given $\frac{S}{a}$, $g_v = A + \frac{B}{\sqrt{a}}$, but the deviations it would lead one to expect are in the opposite direction from those observed. On the other hand, for a given radius, g_v shows a minimum which we find in practice.

In the second case, when *c* is small we have

$$g_v = g_0 + \frac{1}{\sqrt{a}} \left[\frac{2ag_0}{\frac{S}{a}} \left(2f - 1 - \frac{1}{f} \right) \right]^{\frac{1}{2}} \quad . \quad . \quad c3.4$$

This is in quite good numerical agreement with observation on larger spacings, when the values of g_0 and g_v are taken from the appropriate sparkover equation

$$g_v = g_0 + \frac{a}{S} \quad . \quad . \quad . \quad . \quad c3.5$$

(S=spark length=*c* here).

It is also observed that when the critical zones meet in the middle of the sphere gap the arrangement must necessarily act as a uniform field, as is observed in practice.

(For a more complete discussion of the sphere gap see Ref. L/T16.)

(v) *Effect of Pressure and Temperature*

By applying the general discharge theorem to all of these formulæ, which contain only one parameter, viz. the radius of the corona-forming electrode (wire or sphere), as before, we obtain the effect of density, which, as has already been noticed, include the effects both of pressure and temperature. As the formulæ are identical with those considered under Peek's theory (except

that for spheres the constants are different) no more need be said here.

It is interesting to note that Peek obtained the pressure and temperature effects empirically, but his empirical formulæ obey the general discharge theorem.

Further, we can obtain the effect of density in Townsend's theory not only by applying the discharge theorem to the final formulæ for each arrangement, but also from first principles, by considering the effect of density in parallel plate sparking. We have been using the formula for the latter in the form

$$g_v - g_0 = \frac{2{\cdot}7}{c} \quad . \quad . \quad . \quad . \quad c3.6$$

where

g_v = surface gradient,
g_0 = breakdown strength,
c = critical distance.

We can include the effect of density by applying Paschen's law, which transforms the above equation to

$$g_v - g_0\delta = \frac{2{\cdot}7}{c} \quad . \quad . \quad . \quad . \quad c3.7$$

where δ = density.

The case of the wire in a cylinder and parallel wires both give the equation

$$\frac{g_v - g_0\delta}{g_0} = \frac{c}{a} \quad . \quad . \quad . \quad . \quad c3.8$$

For we must remember that the breakdown strength must be proportional to the density.

Thus

$$(g_v - g_0\delta)^2 = \frac{2{\cdot}7 g_0 \delta}{a},$$

or

$$g_v = g_0\delta\left(1 + \frac{0{\cdot}3}{\sqrt{a\delta}}\right) \quad . \quad . \quad . \quad c3.9$$

which is identical with the equation obtained both by applying the general discharge theorem direct and, empirically, by Peek.

(vi) *Low Pressure Effects*

(See also I (*b*) for Experimental Laws)

According to Townsend, sparking takes place over a distance c from the surface of the wire (taking the case of wire and cylinder). Expressing c from the equations already given, in terms of the radius of the wire and the density of the air we have

$$c^2=\frac{2{\cdot}7a}{g_0\delta},$$

where a=radius of wire, g_0=breakdown strength at unit density, and δ=density.

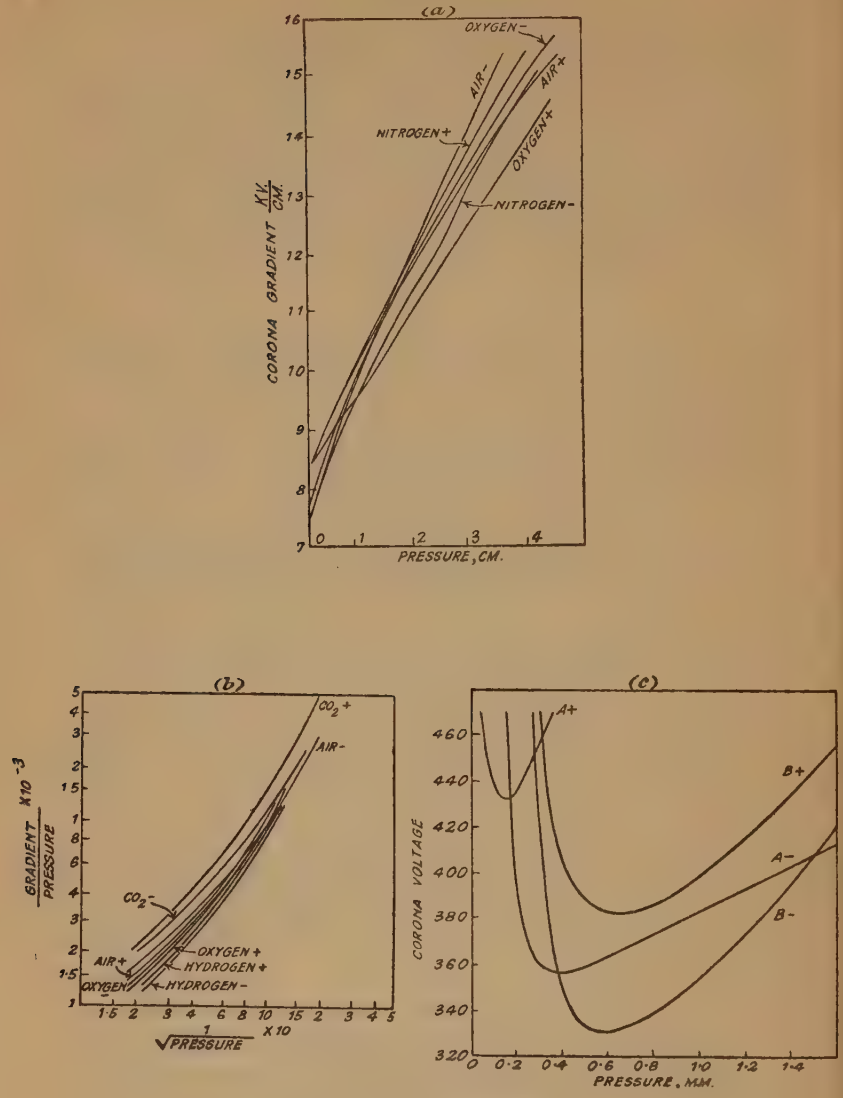
As the density decreases c increases, so that eventually c is greater than A$-a$, where A=radius of cylinder. In other words, the ionisation volume fills the space between the wire and cylinder. If the density is now again decreased, to follow the ordinary corona laws the ionisation must extend beyond the outer cylinder. This is impossible. Accordingly, since a certain current of ionisation is essential for the discharge, the ionisation must be more intense in the smaller space allotted, and in this way a higher voltage is required.

Thus can the main features of Meservey's [95] curves be explained. The minimum of voltage (Diagram XXXIV (*b*)) occurs when the ionisation volume reaches the cylinder, and the voltage afterwards increases. We should expect the minimum for wires of larger diameter to occur at higher pressures than for wires of smaller diameter. This, as is seen on Diagram XXXIV (*b*), is the case. The theory has also been tested numerically in the following manner :—

If the ionisation volume should, theoretically, spread beyond the cylinder, then the force at the cylinder should be greater than the breakdown strength for the given density, *i.e.* greater than 305, or, expressing the pressure in millimetres, greater than $0{\cdot}04p$ (p=pressure), where the stress is $\frac{k\text{V}}{\text{cm.}}$. This is found to be the case from actual experiment.

It must be pointed out that, although the theory describes

DIAGRAM XXXIV



Corona at Low Pressures.

adequately the course of events at low pressures, accurate quantitative values cannot be obtained.

(vii) *Limitations to Townsend's Theory*

In the first place it must be admitted that Townsend's theory fails to account for the difference between positive and negative corona. No such difference occurs in uniform field sparking, and so, on Townsend's theory, none should occur in corona. However, negative corona seems to show some fundamental characteristics different from positive corona.

In the part on Sparking we observed that different experimenters showed a considerable divergence in their results on sparking in uniform fields, when the product, pressure (in cm.) × spark length (in cm.), fell below 100, and, when less than 10, the results were hardly comparable. Accordingly, we cannot expect Townsend's theory to be any more satisfactory when his corona theory demands the application of such results. The effective spark length on his theory is given by $c=\sqrt{34aP}$ (a=radius in cm., P=pressure in cm.); therefore, when Pc, *i.e.* $\sqrt{34aP}$, is less than 100, the theory should show divergence, and cannot be expected to give satisfactory results when $\sqrt{34aP}$ is less than 100.

Lee and Kurrelmeyer [87] have criticised Townsend's theory on the grounds that it was unsatisfactory at low pressures. These experimenters used wire 0·066 cm. in diameter (approx.). Therefore Pe, in this case, is equal to $\sqrt{34\times0{\cdot}033P}$. In Diagram XXXV ($a$) six values for pressures between 10 and 30 cm. are plotted. This gives a range for Pc from 10·6 to 18·35, where we should not expect the results to be very good. The points do, however, give good agreement with a formula

$$g_v \text{ (surface gradient)}=A+\frac{B}{\sqrt{P}},$$

which is of the right form. A and B, the constants, are at any rate of the right order, which, considering the corresponding sparking results, is as much as we can expect.

Thus Townsend's theory can give a good qualitative explana-

tion of the main features of low pressure phenomena, and gives qualitative agreement with experiment, as far as the results for sparking (which he applies) are satisfactory.

(viii) *Field in Corona*

Townsend's assumption, that the field in the actual corona is effectively uniform, is borne out by Booth's [94] results (Diagrams XXXII and XXXIII). The voltage is seen to drop in almost a straight line, showing that the gradient varies slightly and uniformly. However, for larger corona thicknesses, the assumption that we may put the effective gradient as equal to the mean between the surface gradient and the breakdown strength (*i.e.* the gradient at which ionisation ceases) is probably not justified. In most cases which occur in practice, however, the assumption is not very far out.

It must also be remembered that only critical starting gradients are considered, *i.e.* gradients where corona has just formed, but when the current is very small, and so the space charge has a negligible effect. When the currents are larger the space charge modifies the field considerably, and so the theory no longer applies. It is for this reason, probably, that Townsend's theory is unable to offer, so far, an explanation of the relations between corona and the spark which occurs when the voltage is raised too high.

III. EFFECT OF CONDITIONS

(*a*) Difference between Positive and Negative Discharges and A.C. Effects

(i) *Appearance*

As has already been observed, the appearance of negative corona is, in general, quite different from that of positive. In the former case we have a number of beads, spaced more or less regularly, while in the latter we have a uniform glow.

According to Lee and Kurrelmeyer,[87] the beads do not appear until the voltage is raised well above the critical voltage; but these writers agree that the beads, when they do appear, are regularly spaced. Kunz [99] describes the appearance of the discharges in

similar terms. He thinks that the negative beads are an intrinsic property of the negative discharge, and states that they depend on the pressure and applied voltage. He finds that a spark gap in series destroys the difference between positive and negative corona.

Crooker,[101] however, considers that the negative beads are to a considerable extent dependent on the nature of the surface. Thus, using corroded and enamelled wires, the beads were absent, but brushes appeared when the voltage was raised. Lee and Kurrelmeyer [87] worked with carefully polished wires.

Peek [102] also observes that, after use with corona, a negative wire is oxidised all over its surface, except where the beads have been. Here the surface is bright.

(ii) *Differences in Laws of Positive and Negative Corona*

Both positive and negative corona follow laws of the forms already enunciated. There is, however, a certain difference in the values of the constants, which becomes appreciable at low pressures. MacKenzie [96] early found that corona rectifies if used with A.C. This we should expect if corona starts at different voltages for the positive and negative discharges. MacKenzie [96] thinks that this difference depends upon the radius and pressure, becoming zero at certain values. Schaffer [136] agrees that, other things being equal, the ratio $\frac{\text{positive volts}}{\text{negative volts}}$ is a function of the radius.

Lee and Kurrelmeyer [87] have investigated D.C. corona for both positive and negative discharges, and some of their curves have already been given in Diagram XXXIV (*a*) and (*b*). For air, oxygen, and helium, negative corona occurs at higher voltages than positive at the higher pressures, but the curves cross, and at lower pressures the reverse is true. For hydrogen, nitrogen, and carbonic acid, positive corona always requires higher voltages than negative corona, the curves not crossing. At the point of crossing, in the case of air, a break in the A.C. curve appears.

[101] Crooker, *Phys. Rev.*, 8 (1916).
[102] Peek, *Dielectric Phenomena*, p. 77.
[136] Schaffer, *Phys. Zeit.*, **14**, 981 (1913); **15**, 405 (1914).

(iii) *Low Pressures*

As has already been noticed, considerable differences appear between positive and negative discharges at low pressures of a fraction of a millimetre. For thinner wires in air, the curves for positive and negative corona voltages again appear to cross at very low pressures, while for thicker wires the negative curve completely surrounds the positive curve (see Diagram XXXIV (*c*)).[95]

MacKenzie,[96] who found Peek's equation verified down to quite low pressures, states that, at very low pressures, positive corona can always form, but negative corona for larger wires may not appear at all, only a vacuum discharge being obtained. In the case of positive corona at very low pressures it may, if the voltage is raised, be changed into a vacuum discharge.

In this connection we may note that Crooker[101] considers the beads in the negative corona to be analogous to the cathode spot in the vacuum discharge.

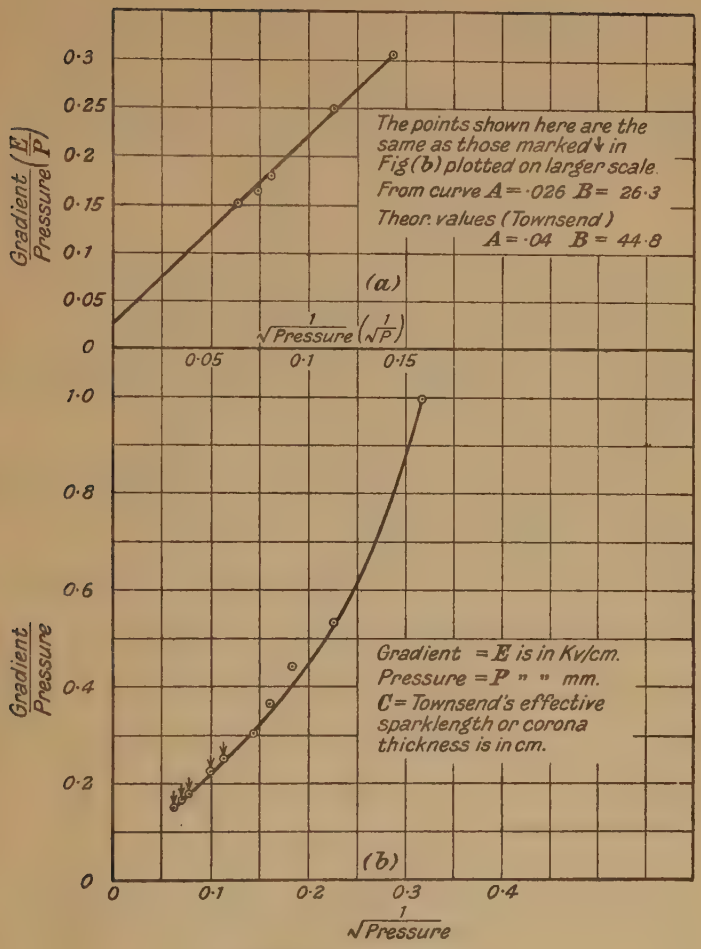
(iv) *Discharges from Points*

According to Warburg[58] and Zeleny,[90] with blunt points positive discharges occur at lower potentials than with negative points. With sharp points the reverse is the case. Again, the distances from other conductors (in this case a plane or cylinder) were not the same for stable positive discharges as for stable negative discharges.

(v) *Townsend's Theory*

In the Introduction, Section III (*d*), a theory was given by Townsend showing that in non-uniform fields a difference between positive and negative sparking is to be expected. This applies equally to corona, provided that the integrals in the theory are taken over only those parts of the field where ionisation by collision is taking place. According to Townsend's theory of corona the field is effectively uniform in these parts, and that is why his theory fails to account for the differences between positive and negative corona. At ordinary pressures the assumption is not far out, and, in fact, the difference between positive and negative corona is not very great; but as the pressure decreases

DIAGRAM XXXV



Corona at Low Pressures.

the corona increases, and the field in the actual corona can no longer be regarded as uniform. Accordingly, we expect and find differences between positive and negative discharges.

In the paragraph mentioned it was shown that the positive discharge will occur at a higher or lower potential according to whether

$$\frac{1}{\beta}\frac{d\beta}{dx} > \text{ or } < \frac{1}{\alpha}\frac{d\alpha}{dx} \quad . \quad . \quad . \quad c4.1$$

where x=mean field strength, α and β=coefficients of ionisation by collision for negative and positive ions respectively; but since α and β depend on the pressure as well as the field, the difference between positive and negative corona will depend on the pressure.

Since, again, α and β depend upon x, and since the field is stronger for small wires than for large wires, the difference between the positive and negative corona will be a function of the radius as well as of the pressure.

If we try a few rough experimental values we find that, where x=field strength and p=pressure, $\frac{x}{p}$ is greater than 150, negative discharges on the whole occur more easily than positive discharges. Thus with blunt points x is small, and therefore $\frac{x}{p}$ is less than 150, consequently positive discharges are easier. For sharp points x is large, so that negative discharges will probably occur at lower voltages, although the voltage for sharp points is less than for blunt ones. We cannot apply this deduction to corona on a wire in a cylinder quantitatively, but from Meservey's curves we see that there is more difference between positive and negative corona for small wires than for large ones—as is to be expected, since the field is stronger. The negative discharge occurs at lower potentials than the positive, in agreement with the theory.

(vi) *Lee and Kurrelmeyer's* [87] *Suggestions*

Lee and Kurrelmeyer have pointed out that part of the difference between positive and negative corona may be due to the

distance through which an ion must travel before attaining its maximum energy. For lower pressures this difference is appreciable, and is different for different ions. As the pressure increases the distance increases, so that the distances for either the positive or negative ion must tend to the same limit with increasing pressure. This explains more or less why the difference becomes small at higher pressures, but it only partially attacks the problem.

(vii) *The Negative Discharge at Low Pressures*

Although the matter will be considered again when discussing the relations of corona with other discharges, it may be pointed out that there is a certain amount of similarity between negative corona and the vacuum discharge. Crooker [101] has suggested the analogy between cathode spot and negative corona beads. In negative corona we have a considerable voltage drop at the wire, *i.e.* the cathode (see Booth's fig. 3, Diagram XXXIII). Similarly in the vacuum discharge we have the characteristic cathode fall of potential. Further, the idea suggests itself that the clean-up produced by the beads (Peek) may be due to the impact of positive ions which, in the case of the vacuum discharge, produce by their impact on the cathode electrons to carry on the discharge. Again, positive corona, according to MacKenzie,[96] can always form, whereas negative corona passes into a vacuum discharge.

(viii) *A.C. Effects*

Since there are differences between negative and positive corona, and since these differences are affected by such conditions as diameter of wire, pressure, nature of surface, etc., A.C. corona will present similar variations.

Thus at certain pressures in air depending on the wire diameter there is a break in the A.C. corona curve. At higher pressures with wire and cylinder, positive corona occurs at lower voltages than negative corona, so that A.C. corona will, on the whole, favour most of the characteristics of positive corona. At lower pressures negative corona will occur more easily, so that A.C. corona will resemble the negative discharge.

With parallel wires carrying A.C., the appearance on each wire

is the sum of the effects of positive and negative corona. If, however, the wires are examined through a stroboscope, adjusted to the correct phase angle, one wire is seen to give positive corona and the other negative. Peek's book, *Dielectric Phenomena in High Voltage Engineering*, should be consulted for the appearances observed, as it contains several photographs superior to the majority of those hitherto published.

If an arrangement is used such that corona only forms round one electrode, the discharge will rectify (as has been mentioned), but if a spark gap is used in series no rectification ensues. Kunz found that a spark in series destroys the difference between positive and negative discharges, and so should also destroy the rectification.

(*b*) Effect of Variable Conditions

(i) *Frequency*

Peek[103] found no important effect due to change of frequency up to 30,000 cycles (see Diagram XXXVI (*a*)), but this is contradicted by Gorton and Whitehead,[104] who state that high frequency slightly lowers the critical voltage. Some interesting experiments might be made to determine whether there is a critical time required for the formation of corona. At the frequency corresponding to this time interval there should be a notable rise in the critical voltage.

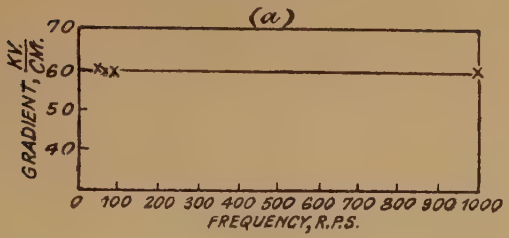
(ii) *Initial Ionisation*

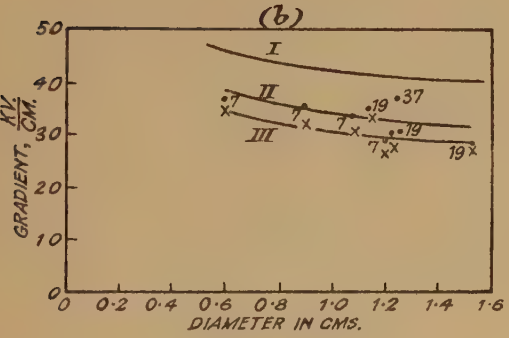
Peek[105] says that the presence of a considerable amount of initial ionisation does not appreciably affect the starting voltage. He does not, however, mention what effect the absence of any initial ions would have. Whitehead[106] has found X-rays to have no effect. Lee and Kurrelmeyer[87] have also obtained no effect from the ultra-violet light given out by an iron arc.

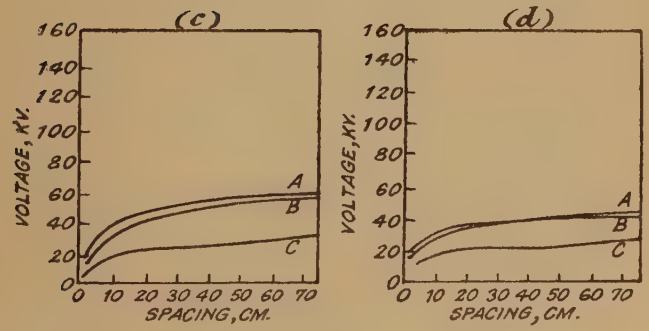
It might be suggested that the initial ionisation necessary to start a normal discharge from wires is small, owing to the

[103] Peek, *Dielectric Phenomena*, p. 65.
[104] Gorton and Whitehead and Ryan, *J.A.I.E.E.* (1914).
[105] Peek, *Dielectric Phenomena*, p. 67.
[106] Whitehead, *A.I.E.E.* (1910).

DIAGRAM XXXVI







A. Normal. B. Oil on Surface. C. Water on Surface.

Effect of Frequency and Nature of Surface on Corona.

comparatively large " critical volume " defined in Zeleny's theory of spark lag. There are bound to be enough casual ions favourably placed. In the case of points, where the critical volume is small, the presence of ionising radiations causes the discharge to begin and end at the same potential.

During the corona process there is a large ultra-violet emission. This should act photo-electrically upon the metal of the electrodes, producing photo-electrons. The importance of this electron emission is still doubtful, owing to the difficulty of separating it from other effects ; but the most general opinion at present is that they are of relatively small importance.

More recently, the importance of initial ionisation on corona formation has been shown by Compton and Foulke.[137] They find that corona is greatly affected by the radioactive radiations from the earth. By the use of pure air and by protecting the discharge gap from the earth's radiation by means of a lead shield, the discharge was started with great difficulty. As it is impossible to remove all ions, it seems that these experiments give strong support to the view that initial ionisation is essential to discharge.

(iii) *Nature of Surface*

We have already mentioned that for the formation of the characteristic negative beads the wire must be smooth. Crooker[101] used a steel wire, one-half of which was polished while the other half was corroded, and found that the beads did not form on the corroded half. This observation is borne out by most experimenters on corona. It may possibly be due to the fact that, at the roughnesses, small local brushes or beads form, which, owing to the high local flux concentration, are stable, and so prevent the development of the regular beads.

In this connection the experiments of Whitehead[107] on stranded cables have a bearing. He found that the corona point was not so sharp, and distinguished a low starting potential with local corona, *i.e.* a partial corona at the parts of greatest intensity. On increasing the voltage the corona spread over the cable, but even then the " full corona voltage " was less than for a smooth

[137] Compton and Foulke, *General Electric Review*, pp. 26–756 (1923).
[107] Whitehead, *A.I.E.E.* (June 1911).

wire of equivalent overall diameter. The effects resemble Crooker's, and the explanation of local discharge seems to fit both (see Diagram XXXVI (*b*) for Whitehead's curves).

The same phenomena is observed with dirt (Peek)[108] or abrasions (Crooker)[101] on the surface, local brushes occurring.

Flux concentration by interposed dielectrics also lowers the corona voltage. Peek[108] has shown that oil and water on the surface lower the starting potential considerably, although in the case of oil the effect of flux concentration seems to be outweighed by the increase of cross-section due to the oil layer. Water shows a very large lowering. (For the effects of oil and water see Diagram XXXVI (*c*) and (*d*).) We should expect a large lowering with water, owing to the high dielectric constant, but it is possible that it also supplies ions as it evaporates.

Crooker[101] found that enamelling a wire lowered the corona voltage, and attributes it to flux concentration by the enamel.

(iv) *Humidity*

Humidity, of course, affects corona phenomena, in so far as it increases the pressure and deposits dew on the electrodes. Such effects have already been discussed above.

According to Peek[109] and Whitehead, humidity has no intrinsic effect apart from the above considerations. It might be expected that, by the formation of heavy ions, it would give variations, but such an effect has not so far been observed with corona.

(v) *Current in Wire.*

If a current is sent through a wire round which corona has formed, the consequent heating will affect the discharge by warming the gas. Apart from this, it might be expected that the magnetic field due to the current would affect the corona by deflecting the ions. Such a magnetic field would have to be large, owing to the small mean free paths of the ions, and according to Peek[109] no such effect appears up to 80 amps. current.

Lee and Kurrelmeyer[87] think that the matter may be important and are about to investigate it; but in view of Peek's results the

[108] Peek, *Dielectric Phenomena*, p. 66.
[109] *Ibid.*, p. 68.

experiments must either be carried out at low pressures (where the free paths are longer), or some device made to annul the heating, before the results can be considered reliable.

IV. SECONDARY PHENOMENA IN CORONA

(*a*) Current in Corona and Power Loss

(i) *Current with Wire and Cylinder*

Both Almy [110] and Watson [82] have found that with corona, as the applied voltage is increased, the current flowing between the electrodes increases also. Their results fit the law

$$i=cv(v-\mathrm{V}) \quad . \quad . \quad . \quad . \quad c5.1$$

where v=applied voltage, V=voltage at which corona begins, c=constant.

Further, according to Almy,[110] the constant c depends upon the radius of the external cylinder being approximately proportional to its inverse cube.

Townsend [111] has obtained a formula theoretically for a positive discharge where the thickness of the ionisation layer is small. The current is given by the equation

$$i=\frac{2\pi qk_1v}{\log\frac{\mathrm{A}}{a}} \quad . \quad . \quad . \quad . \quad c5.2$$

where

v=applied voltage,
k_1=mobility of positive ion,
a=radius of wire,
A=radius of cylinder.

It is assumed that there is a uniform positive space charge of density q outside the corona layer; this latter, owing to its size, exercising an inappreciable effect on the field. This space charge decreases the force at the wire by an amount,

$$\frac{\pi q\mathrm{A}^2}{a\log\frac{\mathrm{A}}{a}}.$$

[110] Almy, *American Journal of Science*, 12, 175 (1901).
[111] Townsend, *Electricity in Gases*, p. 376.

The effective force at the wire is thus

$$\frac{v}{a \log \frac{A}{a}} - \frac{\pi q A^2}{a \log \frac{A}{a}}.$$

Townsend now assumes that this force is the minimum necessary for the discharge, *i.e.*

$$\frac{V}{a \log \frac{A}{a}}$$

where V=minimum voltage for corona.

On substituting from *c*5.2 for q we have

$$v(v-V) = i \frac{A^2 \log \frac{A}{a}}{2k_1} \quad . \quad . \quad . \quad c5.3$$

This formula agrees qualitatively more or less with Almy's results (Almy does not give all the necessary data), and agrees numerically with Watson's results within a few per cent. The formula is not, however, very satisfactory, since we know that the size of the corona increases with the voltage, wherefore the effective gradient at the wire surface must increase also. This is contrary to the hypothesis. For the complete calculation see Appendix VI.

In the case of negative discharges, changes of conditions, such as humidity, etc., have such a large effect on the mobilities of the negative ions (which are most important here) that theories are difficult to construct.

(ii) *Currents from Points*

Warburg [112] gives a similar formula for the curves from points, namely,

$$i = cv(v-V),$$

where, as before, v=applied voltage, V=starting voltage, c=constant.

Zeleny [90] found that this formula represented accurately his results for positive point discharges, but the currents obtained

[112] Warburg, *Wied. Ann.*, **67**, 72 (1899).

with negative points could not be represented by any simple formula.

More recently, Finkelstein-Cukier [113] has criticised Warburg's formula for points. He states that it only holds approximately for air, and even then the air must be changed after each experiment or the measurements made in an air current. He gives

$$i=c(v-V)(v+b)^2 \quad . \quad . \quad . \quad c5.5$$

(where b is a second constant) as being more accurate and of wider application.

(iii) *Relation between Current and Power Loss*

With D.C. corona an amount of power vi, where v is the applied voltage and i is the current, would be dissipated in the gas. The quantity vi would therefore represent the power loss in this particular case. In the case of a wire and cylinder the formula already obtained for the current would give a power loss

$$vi=cv^2(v-V) \quad . \quad . \quad . \quad . \quad c5.6$$

where

v=voltage,
V=starting voltage,
i=current,

and probably a similar equation would hold more or less for points and parallel wires.

However, D.C. power loss has hardly been investigated at all as such, although it seems that it would yield interesting and useful results, particularly for a wire in a cylinder where the field is calculable.

A.C. power loss, which is very important in practice, is a more complicated matter. The current passes only when the voltage is above a certain value, so that the power loss will depend upon the frequency. Further, the matter is complicated by the development of harmonics.

Making the assumption that corona occurred when the voltage rose above V, and using Warburg's formula, we should expect a power-loss equation of the form

[113] Finkelstein-Cukier, *Ann. der Physik*, **71**, 8 (1923).

$$p=4cf\left(\frac{\mathrm{V}m^3}{3}-\frac{\mathrm{V}m^2\mathrm{V}}{2}+\frac{\mathrm{V}^3}{6}\right). \quad . \quad . \quad c5.7$$

where

p=power loss,
f=frequency,
V_m=maximum voltage,
c=constant depending on configuration of the electrodes.

Such a form, however, has not been obtained in practice. In any case, there may be a lag between current and voltage which would introduce a phase angle.

(iv) *Peek's Formula for Parallel Wires*

Peek has investigated power loss between parallel wires with corona at considerable length, and his work is a classical investigation. It is given in full in chap. v of his work, *Dielectric Phenomena in High Voltage Engineering.*

His results are expressed briefly in the formula

$$p=241(f+25)\sqrt{\frac{r+\frac{6}{s}+0{\cdot}04}{\mathrm{S}}}(e-e_d)^2 10^{-5} k\mathrm{W} \text{ per } k\mathrm{M} \text{ of wire} . \quad c5.8$$

where

p=power loss,
f=frequency,
r=radius of wire,
S=spacing,
e=applied voltage,
e_d=corona-starting voltage.

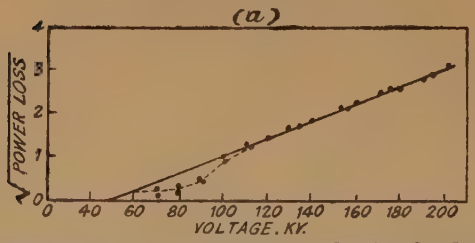
On Diagram XXXVII (a), (b), and (c), some curves are shown giving the variation of p with $(e-e_d)$, f, and $\frac{\mathrm{S}}{r}$ respectively.

The corona-starting voltage does not give as satisfactory a value for e_d as a quantity which Peek calls the disruptive critical voltage, and which he deduces from power-loss measurements. g_d is the corresponding gradient, where

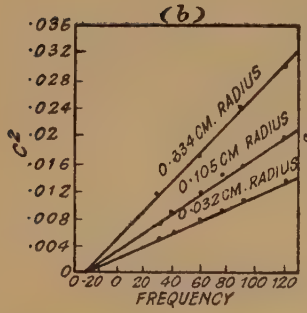
$$g_d=29{\cdot}8\delta\left(1+\frac{0{\cdot}3}{\sqrt{\delta r}}\right) \quad . \quad . \quad . \quad c5.9$$

where δ=air density.

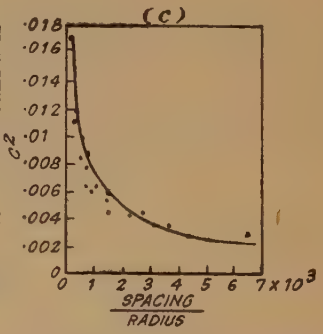
DIAGRAM XXXVII



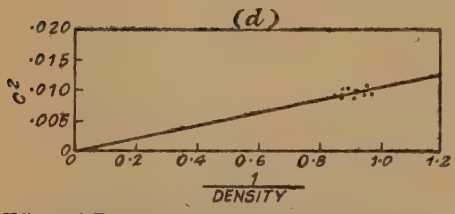
Variation of Power Loss, not Voltage, showing Quadratic Law. Power Loss = Const. (applied Volts − starting Volts)². No. 8 Copper Wire, 0·328 cm. diameter, Spacing = 183 cm. (Peek).



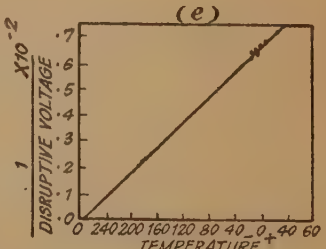
Variation of C^2 with Frequency. C^2 is constant in Equation. Loss = C^2 (applied Volts − starting Volts)² (Peek).



Variation of Power Loss with $\frac{\text{Spacing}}{\text{Radius}}$. C^2 is constant in Equation. Loss = C^2 (applied Volts − starting Volts)² (Peek).



Effect of Density on C^2 where C^2 is the constant in the Quadratic Law. Loss = C^2 (applied Volts − starting Volts)² (Peek).



Effect of Density (*i.e.* Temperature here) on disruptive Voltage (Peek).

The value of e_d is derived from g_d by the usual formula—

$$e_d = g_d r \log \frac{S}{r}.$$

For larger values of the radius r, $g_d = 29{\cdot}8$, *i.e.* is equal to the minimum breakdown strength, while for small values it is practically identical with the corona-starting gradient.

The adoption of this disruptive voltage instead of the starting voltage seems rather a weakness, since the formula suggests that there is a power loss at voltages lower than the corona-starting voltage, which, of course, is absurd. However, the above law does not hold at voltages near the starting voltage.

(v) *Effect of Air Density*

For the same applied voltage and frequency the loss is inversely proportional to the air density, according to Peek.[133] This correction is not very satisfactory, as only a small range has been examined, as is seen in Diagram XXXVII (*d*). The disruptive critical voltage e_d (in the equation) is, according to Peek, proportional to $\frac{1}{\text{density}}$ (Diagram XXXVII (*e*)). On Diagram XXXVIII (*a*) is given a diagram showing the relation according to Peek between the disruptive critical gradient, the starting gradient for corona, and the minimum breakdown strength.

(vi) *Power Loss at Voltages near Starting Voltage*

When the voltage approaches its minimum value for corona, the above law, known as Peek's "quadratic law," no longer holds; the measured loss being greater than that given by Peek's equation for large conductors, while for small conductors the measured loss is less than the theoretical.

For large conductors Peek explains this divergence by the statement that local brushes, etc., occur at irregularities, and so increase the loss. He supports this by pointing out that, if to the theoretical loss is added a probability term $be^{-n(e-e_d)^2}$ (e=voltage, e_d=disruptive critical voltage), which represents the effect of a probability distribution of irregularities as far as

[133] Peek, *Dielectric Phenomena*, p. 27.

voltage is concerned, the result is the measured curve. This is shown in Diagram XXXVIII (*b*).

For small conductors the irregularities are comparable with the shape of the wire itself, so that they have little specific effect. This does not explain, however, why the measured loss should be less than that given by the quadratic law. The effects are shown in Diagram XXXVIII (*c*).

(vii) *Power Loss at High Voltages*

At high voltages the quadratic law is unsatisfactory. Harding[114] has made measurements with very high voltages, and finds that the quadratic law gives values for the power loss which are too high. According to him, at these voltages, the relation between the power loss and the difference between applied voltage and starting voltage ($e-e_0$) is linear. Some of his results are shown in Diagram XXXVIII (*d*).

(viii) *Effect of Weather Conditions on Power Loss*

Peek has given results taken under all kinds of weather conditions, and for such details his book, *Dielectric Phenomena in High Voltage Engineering*, chap. v, must be consulted.

He finds that moisture increases the loss considerably. During a snowstorm the loss was 2 or 3 times the fair-weather loss. The effect of snow and sleet, however, decreases as the voltage is raised. Rain has a very strong effect, glows turning into brushes when the wires become wet.

Wind has no effect, but smoke produces a slight increase of loss.

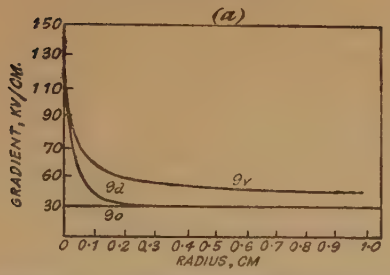
(ix) *Effect of Frequency on Power Loss*

High frequency appears to decrease the power loss at high voltages, but the data are not very considerable. Some of Peek's results are shown in Diagram XXXVIII (*e*).

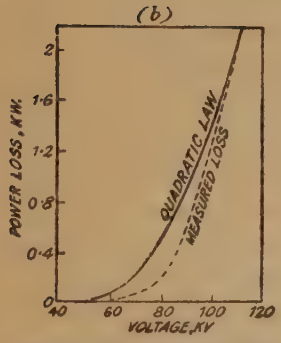
(x) *Anderegg's Reversal*

When corona has formed there is a production of ozone and other gases from the air. This gives rise to a change in pressure.

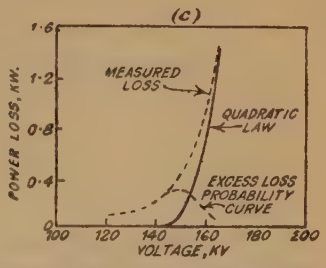
[114] Harding, *J.A.I.E.E.* (October 1924).



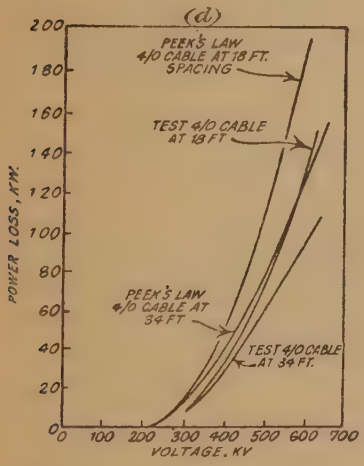
Comparison of g_o Surface Gradient for Corona, g_d Disruptive Gradient from Power Loss, g_o Breakdown Strength.



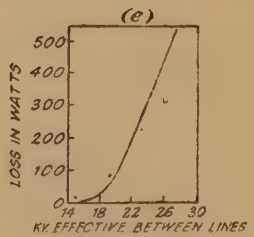
Power Loss near starting Voltages (Peek). Small Conductors No. 8 Copper Wire, 0·328 cm. diameter, 127 cm. Spacing.



Power Loss near starting Voltages for large Conductors (Peek). 3/0 7-strand cable, 1·18 cm. diameter, 310 cm. Spacing. The measured Loss is equal to sum of Excess Loss and Quadratic Loss.



Departure from Quadratic Law of Power Loss at High Voltages (Harding).



Corona Loss at 10^5 R.P.S. Curve calculated for Quadratic Loss. Point measured (Alexanderson).

Anderegg [115] has shown that, if corona is run for some time, the above-mentioned pressure shows a reversal, *i.e.* at a certain stage it shows a sudden fall followed by a sudden rise. The power loss also shows a similar reversal at this point. According to Anderegg the effect is due to adsorption and subsequent desorption. A surface layer of high dielectric constant is formed and then destroyed.

This matter will be considered in more detail in conjunction with the phenomenon of corona pressure.

(xi) *Hysteresis in Corona*

Many experimenters have suggested that there is a lag of corona behind applied voltage, which with A.C. gives a hysteresis effect.

Lee and Kurrelmeyer [87] have examined the currents in corona between parallel wires in the neighbourhood of corona formation, and find that the current curves for descending and ascending voltage are not identical. A small hysteresis effect seems proved here. Gardner [117] thinks that we have hysteresis with A.C., and Ryan and Henline [123] support this with oscillogram studies.

There is, however, no appreciable difference between A.C. and D.C. starting voltages, so that the starting of corona does not seem to lag very much behind the voltage. This has already been discussed, and depends upon the time required for corona to form. This latter will be better understood when corona has been examined at radio frequencies.

(*b*) Corona in Circuits

(i) *Diameter of Corona*

In this connection we must distinguish between the terms "diameter of visual corona," Peek's "equivalent coronal conductor," and the "effective conductor diameter" deduced from capacity measurements. The first is considered under the heading of the present paragraph.

[115] Anderegg, *Journ. American Electro Chemical Soc.* (1925).
[117] Gardner, *J.A.I.E.E.* (August 1925).
[123] Ryan and Henline, *J.A.I.E.E.* (1924).

Visual effects occur, for practical purposes, coincidently with the ionisation causing the corona ; for the absorption of energy in ionisation and its subsequent emission as radiation effectively coincide. Accordingly, we obtain a measure of the ionisation film by measuring the visual corona, provided, of course, that all radiations (*e.g.* ultra-violet rays) are included.

Whitehead [116] has measured corona diameters for increasing voltages by photographic methods. These methods are difficult to carry out, and not very reliable. The question whether or not the presence of ultra-violet light influenced the phenomenon was left unanswered. Some of his results are given in Diagram XXXIX (*a*). From them it appears that corona starts with a definite thickness, as is to be expected from theory. For a 0·233 cm. wire, according to his results, the minimum corona diameter is about 0·3 or 0·4. Townsend's theory (and Peek's also) gives 0·44 cm. as the minimum diameter.

It would be interesting if further measurements were made with a view to settling this matter.

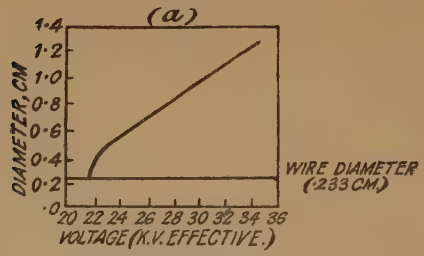
(ii) *Effective Conductor Diameter of Corona*

In an ordinary transmission line we may represent the effect of corona by a capacity to earth with an ohmic leakance to earth in parallel. The ohmic leakance gives the power loss which has already been discussed. Of course, from a consideration of the laws of power loss, and the fact that the leakance only operates when the voltage rises above a certain value, we see that this representation is arbitrary and limited in application. However, for the ranges of voltages over which a line normally operates, the assumption represents the results sufficiently accurately for engineers.

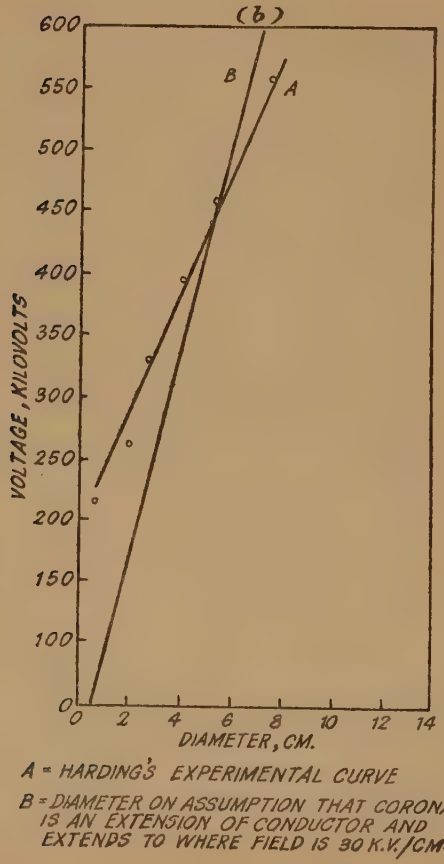
The effective capacity of the assumed equivalent system is found to suggest that corona increases the capacity. Accordingly, Peek has put forward his view (which has already been criticised) of corona as an effective extension of the conductor, which would account for an increase of capacity. Harding has measured this equivalent diameter in the above way (viz. from effective capacitance). His results do not agree very satisfactorily with

[116] Whitehead, *J.A.I.E.E.* (June 1912).

DIAGRAM XXXIX



Diameter of Corona from Photographic Measurements.



Effective Conductor Diameter from Capacity with Corona.

theory. In the curve of diameter *versus* voltage on Diagram XXXIX (*b*) Peek's theory gives a theoretical straight line V=86·52*r* (V=voltage, *r*=radius of corona). Harding's curve is linear, but does not pass through the origin, and has a slope very different from 86·52 (see Diagram XXXIX (*b*) above).

Finally, Gardner [117] has shown that corona may be imitated by connecting a line to earth at various points with pairs of triodes in parallel, so controlled that when the voltage rises above a certain value the valves conduct. In this way he shows that all the observed effects can be imitated without any film of ionisation round the wire. The increased capacity is thus a leakage phenomenon.

Gardner [117] also shows that the increased capacity is due to the method of measurement. The effect of corona is to distort the voltage waves, as it operates only over the peak. If we assume that the wave has a sine form, there results an error which gives an increased measured capacity, as, due to the harmonics, the component of current in quadrature is actually less than must be allotted to it if we assume a sinusoidal wave form.

(iii) *Harmonics due to Corona*

The distortion which corona introduces may be both a source of trouble and also of use.

According to Peek,[118] the chief harmonic developed is the third. This may also be observed from the wave-form studies by Whitehead and Inouye [119] and Bennett.[120] In a three-phase system with an earthed neutral this harmonic can flow through the wires in parallel and complete the circuit through the earth. It may so cause trouble in neighbouring circuits (particularly telephone and telegraph). If the neutral is not earthed, the harmonic appears in each line to the neutral, and the whole system (the voltages being in phase in the different wires) pulsates to earth.

On the other hand, a high-voltage impulse is speedily moderated. The peak is quickly modified, and the whole impulse drawn out by the formation of corona.

[118] Peek, *Trans. A.I.E.E.*, p. 1155 (1921).
[119] Whitehead and Inouye, *A.I.E.E.*, p. 138 (1922).
[120] Bennett, *Trans. A.I.E.E.*, p. 1787 (1913).

Whitehead [121] has recently made some interesting experiments in this connection on the value of corona as a lightning arrester. The energy loss due to radiation is very small indeed until high frequencies are reached, when it rises suddenly to a large amount. The energy loss due to corona is, however, considerable over the whole range of frequency, right up to the point when radiation becomes effective. In this way, if corona can be formed, a line is protected against lightning impulses of whatever frequency may develop ; for either the energy loss due to corona, or that due to radiation, will damp them out before damage has been done.

(iv) *Corona Voltmeter*

The regularity of the corona laws has led to the development of a voltmeter depending upon them. This " corona voltmeter " is described in papers by Whitehead and Isshiki,[122] and by Whitehead and Inouye.[119]

(*c*) Pressure in Corona

(i) *Early Work*

Since ozone and other gases (chiefly oxides of nitrogen) are formed during discharge, we expect certain changes in pressure to accompany them. Further, owing to the increase of energy due to the velocities acquired by the ions, pressure effects are to be expected from this cause. In other words, the energy dissipated in corona may appear, not only as heat with its accompanying effects, but directly as a pressure effect due to ionisation and chemical changes.

Kunz,[99] and also Farwell,[124] first observed this pressure increase and attributed it to the above-mentioned causes. Tyndall and Searle,[125] however, argued that the effect observed was due to the heat developed in the corona.

Warner [126] had, previously to Tyndall and Searle's publication,

[121] Whitehead, *J.A.I.E.E.* (October 1924).
[122] Whitehead and Isshiki, *J.A.I.E.E.* (May 1920).
[124] Farwell, *A.I.E.E.*, **33**, 717.
[125] Tyndall and Searle, *Phil. Mag.*, **35** (1918).
[126] Warner, *Phys. Rev.*, **8** (1916).

found that the pressure increase bore a linear relation to the current when he used positive corona with a wire in a cylinder. He has now repeated his experiments and shown that, although there was a considerable heating effect, it was quite distinct from the phenomenon described by Kunz. Some of Warner's results are shown in Diagram XL (*a*). Kunz's view was also supported by Arnold.[127]

(ii) *Fazel's Experimental Laws*

More recently Fazel[128] has made a careful investigation of the matter, using optical and photographic methods for determining the current and pressure. He has thereby been enabled to observe variations of very short duration. He found that the pressure rise due to heating (in some control experiments) was slow and uniform, as shown in Diagram XL (*b*). Immediately on switching in the corona there was a sudden increase of pressure, followed by a decrease (Diagram XL (*c*)). Subsequently, there was a further increase due to the heating of the corona current. These effects are shown in Diagram XLI (*a*). He then examined the pressure effects carefully both for A.C. and D.C., coming to the following conclusions :—

(1) There is a small pressure difference between the wire and cylinder which dies away quickly.

(2) There is a small pressure increase, lagging behind the corona by about 1·5 secs., which is proportional to the corona current (*cf.* Warner).

(3) With A.C. there is an oscillating pressure increase, having a frequency twice that of the applied voltage. This effect is found superimposed in effect (2) as a datum line increase.

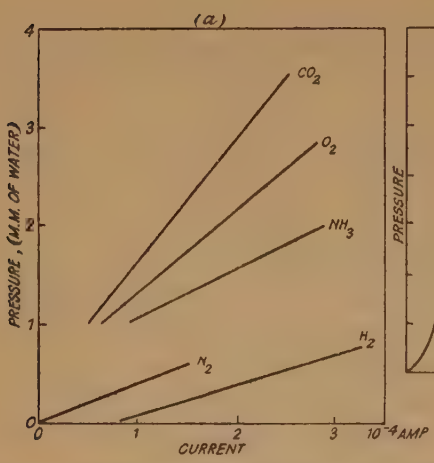
(4) There is a decrease of pressure immediately after the initial increase.

(5) There is a relatively slow subsequent pressure increase due to heating.

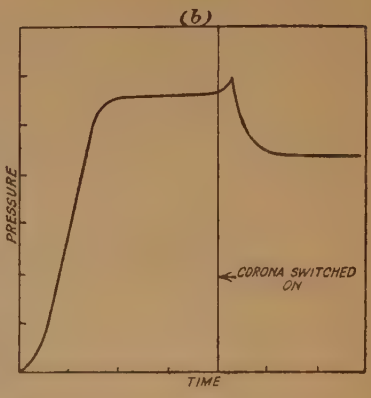
The reason why Tyndall and Searle[125] did not observe the effect was that their measuring instruments were too slow, and indicated only the slow heating effect. As an example of the magnitude of

[127] Arnold, *Phys. Rev.*, **10** (1917).
[128] Fazel, *Phys. Rev.*, **19** (1922).

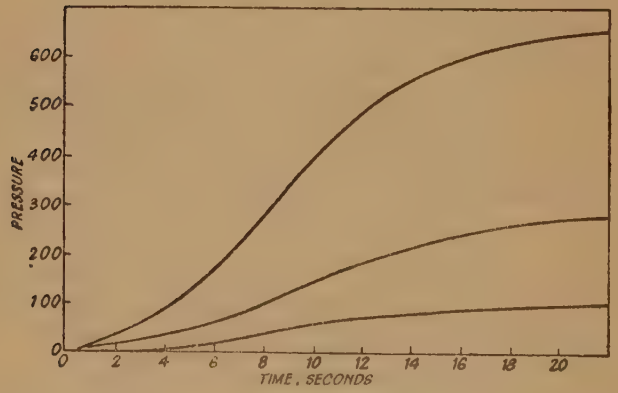
DIAGRAM XL



Variation of Pressure Rise with Current (Warner).

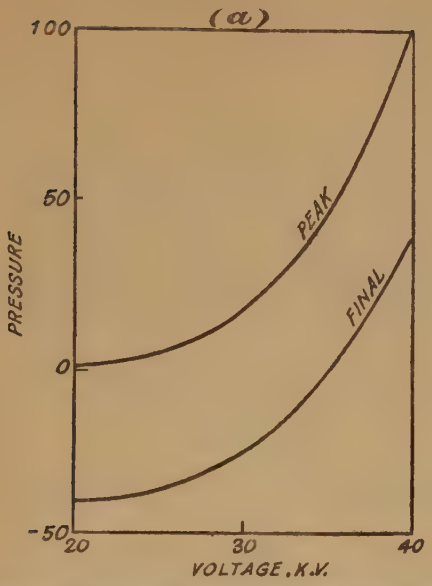


Variation of Pressure with Time (short Intervals) (Fazel).

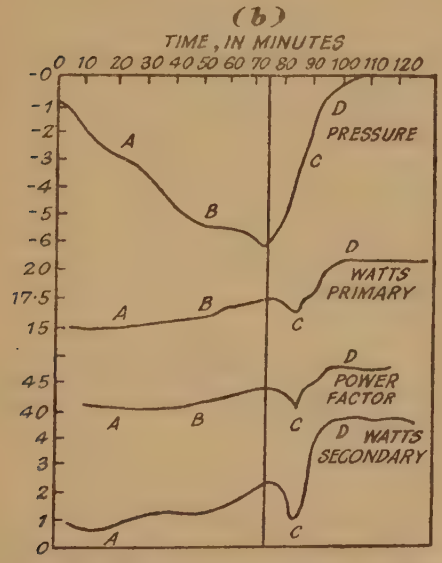


Slow Increase of Pressure with Time for different Currents (Fazel) due to Heating Effect of Current.

DIAGRAM XLI



Variation of Maximum and Final Pressures in Corona and Voltage (Fazel).



Pressure Reversal with Time and simultaneous Effects on Power Loss (Anderegg).

these pressures, for oxygen the pressure increase$=3{\cdot}981\ \frac{\text{dynes}}{\text{cm.}}$, for a current $\frac{1}{12}\times 10^{-4}$ amps. per cm.

(iii) *Kunz's Theory of the Effect* [129]

We can see at once that there must be a linear relation between the pressure increase and the current; for, if the volume and temperature of the gas are kept constant, the amount of energy dissipated by a small change, di, in the current, for an increase of voltage ΔV above the starting voltage, is $\Delta V di$, and this must give rise to an increase in the energy of the gas, $d(pV)$, where p=effective pressure and V=volume.

$$\Delta V di = v d(p_i - p_0),$$

where p_i=pressure with, and p_0=pressure without, corona.

$$\therefore \quad i = \frac{v}{\Delta V}(p_i - p_0) + \text{const.} \quad . \quad . \quad . \quad c6.1$$

Kunz,[129] however, has worked out a complete theory, which is given in Appendix VII. He assumes that the ions give their momentum to the gas molecules, and that no appreciable number of gas molecules is lost to the walls, etc. He obtains the formula

$$\Delta P = \frac{I}{2\pi u}\left[\left(\log A - \frac{A^2 \log A^2 - a^2 \log a^2}{A^2 - a^2} - 1\right)\right]$$
$$= \frac{I}{4\pi u} \text{ (approx.)} \quad . \quad . \quad . \quad . \quad . \quad c6.2$$

in the case of the wire and cylinder, where

ΔP=pressure increase,
I=corona current,
u=mobility of ion concerned (positive for positive discharge, negative for negative discharge),
A=radius of cylinder,
a=radius of wire.

He now explains Fazel's results as follows:—

(1) The pressure between wire and cylinder is due to an ionic

[129] Kunz, *Phys. Rev.*, **19** (1922).

wind, *i.e.* a pressure due to the greater momentum of the ions in the intenser parts of the field.

(2) The pressure increase proper is ΔP, is obtained as above, and follows Fazel's law, as is shown theoretically.

(3) In A.C. the heavy ions are not much affected by the alternations only and give the datum line effect. The light ions, however, take up and give back momentum every half-cycle, and so give the oscillating pressure of double the frequency of the applied voltage.

(4) The heating effect due to the current itself is a simple Joule effect.

(iv) *Mobilities from Pressure Measurements*

Not only does Kunz's theory explain the main experimental results, but also we may calculate, from the values obtained from the pressure effect, the mobilities of the ions concerned (see Appendix VII). Kunz has in this way calculated the mobilities of several gas ions, and compared the values with those obtained by Hess and Chattock, who used direct methods of measurement.

Gas.	Hydrogen.	Nitrogen.	Carbonic Acid.
Mobility by Kunz .	$1{\cdot}51\times10^{-2}$	$0{\cdot}505\times10^{-2}$	$0{\cdot}161\times10^{-2}$
,, ,, Hess .	$3{\cdot}73\times10^{-2}$	$1{\cdot}08\times10^{-2}$	$0{\cdot}45\times10^{-2}$
,, ,, Chattock	5·4		$0{\cdot}83\times10^{-2}$

Gas.	Oxygen.	Ammonia.
Mobility by Kunz. . .	$0{\cdot}23\times10^{-2}$	$0{\cdot}311\times10^{-2}$
,, ,, Hess . . .	$1{\cdot}98\times10^{-2}$	
,, ,, Chattock . .	$1{\cdot}3\times10^{-2}$	

His results are seen to be of the right order, although about one-half the direct values due to Hess. This agreement is satisfactory, considering the nature of the phenomenon.

In the same way Kunz has calculated the proportion of ab normally heavy ions (*i.e.* "Langevin ions") in the discharge. The proportion comes out to about 13·4 per cent.—a likely value.

Kunz has calculated the amount of lag of pressure effect behind corona development, and obtains values of order comparable with Fazel's measurements (namely, about 1·5 secs.).

(v) *Pressure Reversal*

As has already been mentioned, Anderegg [115] has found that, after running corona a little while, the pressure curve has a kink in it, *i.e.* it decreases, and then suddenly increases again. This effect is illustrated on Diagram XLI (*b*). At the same time there are corresponding kinks in the wattage and power-factor curves, as shown in Diagram XLI (*b*).

Anderegg attributes the phenomenon to surface effects. In his paper he shows by preliminary experiments that the amounts of ozone, nitric acid, and ammonia produced are considerably influenced by the surfaces presented in the apparatus.

He goes on to explain the reversal as follows :—

Ozone is first formed and then nitric anhydride. We then obtain heavy ions, which decrease the conductivity and produce the first drop in power factor. These heavy ions are quickly absorbed by the surfaces, and give the kinks A and B in the curves (Diagram XLI (*b*)). The subsequent rise in the current is due to the flux concentration by the adsorption layers, which are of high dielectric constant.

The oxides then decompose the ozone, and the resultant desorption predominates over the adsorption, with the result that the pressure suddenly rises, giving the reversal effect (marked by the line). This evaporation of heavy molecules slows up the ions, and gives a sharp drop in power factor at C. After this (when the heavy molecules are disappearing) there is a steady rise of each of the curves to its own point D.

The theory of adsorption layers is certainly interesting, but the whole system is rather vague at present. It does, however, seem worth while to take parallel readings of the power factor and pressure, as, owing to the empirical state of knowledge regarding power loss, all co-ordinating observations are of value.

(*d*) Gases in Corona (in Air)

(i) *Ozone*

Ozone is one of the most important gases formed during the discharge. As it is an active modification of oxygen it may attack the material of the apparatus. As far as metals are concerned the film of oxides formed protects the metal from further oxidation, but it may produce serious deterioration of rubber and all organic stuffs.

Rideal and Kunz [130] have measured the distribution of ozone in corona in air, and, as is to be expected, come to the conclusion that the ozone molecules are positively ionised. With a negative wire in a cylinder, accordingly, a nearly uniform concentration is found, but with the wire positive the ozone is repelled and accumulates away from the wire.

(ii) *Oxides of Nitrogen*

Nitrogen oxides, chiefly N_2O_4 and N_2O_5, are formed in corona, and may attack materials in the neighbourhood. This is a well-known phenomenon. These in the presence of water give nitric acid (HNO_3), which is observed in thunderstorms.

(iii) *Ammonia*

If hydrogen or hydrogen-forming gases are present, ammonia (NH_3) is formed with corona in air.

V. RELATIONS OF CORONA WITH OTHER DISCHARGES

(*a*) Vacuum Discharge

(i) *Positive Corona*

The characteristic of the vacuum discharge is that there is a fall of potential at the cathode, which enables the positive ions to produce electrons by collision with the cathode.

In positive corona the strong field is at the wire or positive

[130] Rideal and Kunz, *Journ. of Phys. Chem.*, **24** (1920).

electrode, so that we should not expect positive corona at low pressures and the vacuum discharge to be continuous.

However, if at low pressures the voltage is raised in positive corona to the point where sparking should normally occur, a vacuum discharge will result if the pressure is low enough. This is due to the fact that sparking is possible only where sufficient ions can be formed. At low pressures, therefore, the spark cannot form, but instead we have the vacuum discharge. Since the spark tends to be symmetrical, as far as positive and negative discharges are concerned, we can understand why, at the sparking potential, the inhibiting effect of the normal field configuration in positive corona should be removed.

The above observations are borne out by MacKenzie's [96] experiments, which have already been mentioned.

(ii) *Negative Corona*

Since in negative corona the strong field is at the wire or cathode, there is a distinct similarity with the vacuum discharge, and at low pressures they should be continuous. As has been mentioned, MacKenzie [96] found that, although positive corona formed at all the pressures he employed, negative corona was, below certain pressures, replaced by a vacuum discharge.

It is, however, a little difficult to account for Meservey's [95] observations on larger wires (Diagram XXXIV (*c*)). Here he finds that the negative corona voltage curve surrounds the positive corona curve, so that negative corona should be obtained more easily than positive corona. However, very possibly the discharge Meservey measured was actually a vacuum discharge. It would be interesting to see whether the voltage curves for negative corona and the Geissler discharge were continuous.

We have already mentioned Crooker's [101] analogy between the negative beads in corona and the cathode spot in vacuum discharge. Possibly a few electrons may be formed in corona by the collision of positive ions with the wire, but at ordinary pressures the effect must be small. The regular bead spacing may be due to mutual repulsion if the currents are largely carried by ions of one sign.

(*b*) Dark Discharge

(i) *Connection with Corona*

A discharge may take place (*i.e.* a current may pass) in air without visual effects being produced. This is known as the dark discharge. It only occurs in fields where the intense stress extends over a small volume, *e.g.* points.

Although little is known about dark discharges, such experiments (*e.g.* those of Warburg [58] and of Zeleny [57]) as have been made seem to indicate that they obey the same law as corona.

The following explanation may be suggested :—

When the ionisation volume, which forms, as in corona, on the initiation of the discharge, is very small, then the ensuing radiation may be insufficient for visual effects. The discharge, however, is actually of the same kind as corona, and when the voltage is increased, and the ionisation volume thereby extended, visual corona appears, as we should expect.

The currents in the dark discharge are comparatively small.

(ii) *Properties*

There is quite a considerable emission of ultra-violet light, although no perceptible emission occurs in the visible region. Probably, if ultra-violet light photographs were taken, their appearance would be similar to corona photographs taken by ultra-violet light.

Ozone and other gases which are formed in corona in air are also formed in the dark discharge.

The dark discharge is not, however, owing to its comparative infrequency (it tends to pass quickly into glows or brushes) and its small current, very important practically.

(*c*) Brushes

(i) *General*

Brushes can be conveniently classed as over-voltage phenomena in corona. They may also be described as transition phenomena between corona and spark.

They occur when the voltage in corona is raised, brushes

appearing on the irregularities of the corona-forming conductor. In appearance they are sparks or luminous lines, which proceed from some point or irregularity, then branch out, and end diffusely in the air. As the voltage is further increased the brushes become longer, until finally a spark shoots across.

Brushes sometimes form without apparent corona, *e.g.* with sharp points, but if carefully examined a small glow can generally be distinguished.

Brushes are intermittent in character, *i.e.* they resemble sparks, in so far that the current passes in jumps, and, in the case of points (Zeleny,[57] Warburg [58]), they have produced sounds in a telephone receiver in the current circuit.

(ii) *Resemblance to Sparks*

Most of the properties of the brush can be summed up in the statement that it consists of a number of sparks with relatively short paths ending in air. To this extent the brush shows the characteristics of sparking. The current is intermittent, as we have seen, and the brush is both variable in form and unstable generally. In this latter respect it is midway between the glow and the spark.

Brushes probably arise through local flux concentrations, which produce partial sparking. Ions acquire such velocities, and are formed in such quantities in a very limited part of the field that a spark starts and travels for some distance before the weakness of the main field stops it. It then breaks up into portions which are able to cause long paths of ionisation, but which eventually come to an end. Brushes form more easily in negative than in positive corona, owing to the greater mobility of the electrons, which enables them to form long paths of ionisation.

According to Yoshida and Hiratu,[131] the spectra of brush and glow are identical.

(iii) *Tœpler's Observations*

Tœpler [132] observes that in measuring photographically some high-tension brushes, he found that the luminous threads were

[131] Yoshida and Hiratu, Kyoto College of Science, Mem. 5, 2 (1921).
[132] Tœpler, *Ann. der Physik*, **5**, 3 (1917).

of constant thickness in the negative discharge, and that their length was proportional to the discharge potential. The central brush had also a diameter in linear relation with the discharge potential.

(*d*) Spark

(i) *Experimental Effects*

When the voltage in corona is sufficiently raised sparking takes place. The amount by which the voltage must be raised depends upon the configuration of the field.

When the curvature of the field electrodes is sufficiently small compared with their distance apart (*e.g.* for parallel wires and spheres at small spacings, and for concentric cylinders the diameters of which differ by little) only a spark occurs. Here, as we have seen, the spark voltage and gradient follow the extrapolated corona law for the particular configuration employed.

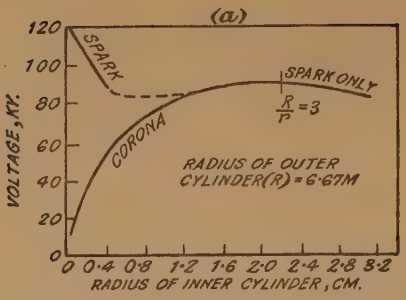
Brushes turn comparatively easily into sparks, and, in general, the formation of brushes is intermediate between corona and sparking.

On Diagram XLII (*a*), (*b*), and (*c*) are illustrated the relations between sparking and corona for concentric cylinders, wires, and spheres respectively.

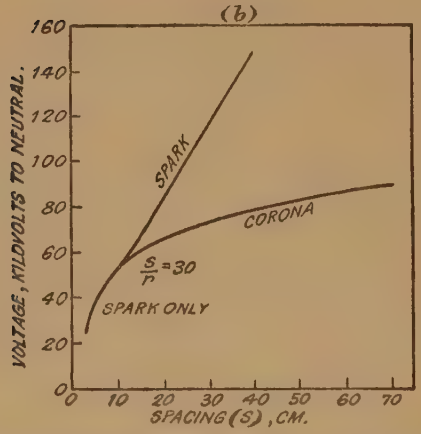
(ii) *Peek's Theory*

Peek [133] states that, when an increase in the size of the corona-forming conductor would increase the gradient, sparking only occurs. He considers corona as an extension of the conductor, and so, if such an extension increases the gradient, the breakdown will proceed cumulatively until the gap is bridged, as in the spark. Experiment does not bear this out. The critical ratio of diameters of concentric cylinders should be 2·7 approximately. Actually the ratio is found to be less for small wires than for large $\left(\text{taking the ratio } \frac{\text{wire}}{\text{cylinder}}\right)$. For parallel wires the critical ratio $\frac{\text{spacing}}{\text{radius}}$ should be 5·85, but in practice it is about 30. We have already seen, from considerations of the field in corona and

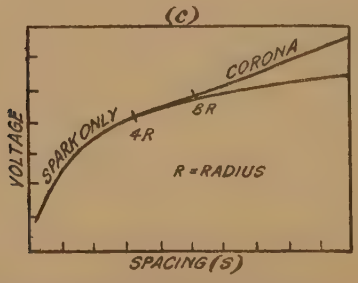
[133] Peek, *Dielectric Phenomena*, p. 27.



Spark and Corona Voltages for Concentric Cylinders.
Critical Ratio on Peek's Theory $\frac{R}{P}=2\cdot718$.



Spark and Corona Voltages for Parallel Wires. Diameter 0·825 cm.
Peek's Theory gives Critical Ratio $\frac{S}{P}=5\cdot85$.



Spark and Corona Voltages for Spheres. Peek's
Theory gives Critical Ratio $\frac{S}{R}=2\cdot04$.

its capacity effect, that the coronal conductor view is not altogether satisfactory. It does not seem to be very adequate here. Peek[134] has amended the equivalent conductor theory by supposing that corona provides a resistance. It is, however, difficult to see the exact application of this, nor does it seem that it provides a better approach than the direct consideration of ionic currents which, owing to ignorance as to ionisation coefficients, mobilities, etc., cannot be accurately dealt with.

(iii) *Critical Path Length*

Another view is that space charges enter into the matter to a large extent. If the part of the distance between the electrodes over which ionisation takes place (the path length of ionisation) is a sufficient fraction of the whole, the space charges developed and the ionic energy formed give the discharge sufficient momentum to bridge the gap, becoming a spark. This, although very vague, is more in accordance with the general principles of sparking.

As the voltage is raised in corona, one tends to obtain long streamers and brushes due to abnormal local ionisation and probably abnormal local mobilities. If these paths of ionisation become long enough, they will become unstable and spark across. In the neighbourhood of the critical ratio it is very difficult, on this account, to distinguish between corona and spark.

Again, we can understand why sparking should obey corona laws where sparking only occurs. The initial voltage need only be sufficient to produce the critical ionisation path; the discharge jumps the rest itself. Accordingly, the voltage corresponds to the formation of corona, which in these cases is unstable.

As we have seen in Sparking, III (*a*) (ii), this view may be expressed mathematically by the continuity condition (see Introduction, III (*c*), equation *a*3.3). In corona and brushes the space charge enables this to hold, providing the voltage is not too high above the critical corona voltage nor the path length too long compared with the gap length. If either of these contingencies occurs the glow or brush is unstable and becomes a spark.

[134] Peek, *Dielectric Phenomena*, p. 85.

By taking into account the finite size of corona, and by making assumptions about its resistance and consequent effect on the field, Peek's theory leads to the above considerations also. It does seem, nevertheless, that to bring in an analogy such as the corona conductor is not justified here. It would be better, one would think, to defer such artificial simplifications until the subject is better understood.

APPENDICES

I. Townsend's Continuity Theorem

Let n_1=number per c.c. negative ions,
n_2=number per c.c. positive ions,
u=velocity of negative ions,
v=velocity of positive ions,
α=ionising coefficient of negative ions,
β=ionising coefficient of positive ions.

If the discharge is to maintain itself, $n_1 n_2$ must not change with time.

$$\therefore \quad \frac{\partial n_1}{\partial t}=-\frac{\partial}{\partial x}(n_1 u)+\alpha n_1 u+\beta n_2 v=0,$$

$$\frac{\partial n_2}{\partial t}=-\frac{\partial}{\partial x}(n_2 v)+\alpha n_1 u+\beta n_2 v=0.$$

If ce=current where e=unit electronic charge,

$$n_1 u+n_2 v=c, \quad \therefore \quad n_1 u=c-n_2 v.$$

Substituting for $n_1 u$, we have

$$\frac{\partial}{\partial x}(n_2 v)+\alpha(c-n_2 v)+\beta n_2 v=0,$$

i.e.

$$\frac{\partial}{\partial x} n_2 v+n_2 v(\beta-\alpha)+\alpha c=0,$$

$$\therefore \quad n_2 v e^{\int_0^x (\beta-\alpha)dx}=-\int \alpha c e^{\int_0^x (\beta-\alpha)dx} dx+\text{B (a const.)}.$$

Let

$$f(x)=e^{\int_0^x (\alpha-\beta)x}.$$

Then

$$n_2 v=\text{B}f(x)-cf(x)\int \alpha[f(x)]^{-1}dx.$$

At the negative electrode, $x=0$ and $n_1=0$, since there are no ions from the electrode. $\text{B}=c$ for $f(x)=1$ and $n_2 v=c$.

At the positive electrode, $x=e$ and $n_2=0$, for there are no ions from the positive electrode.

$$\therefore \quad 0=ce^{\int_0^l (\alpha-\beta)dx}-ce^{\int_0^l (\alpha-\beta)dx}\int_0^l \alpha e^{\int_0^l (\beta-\alpha)dx} dx,$$

i.e.

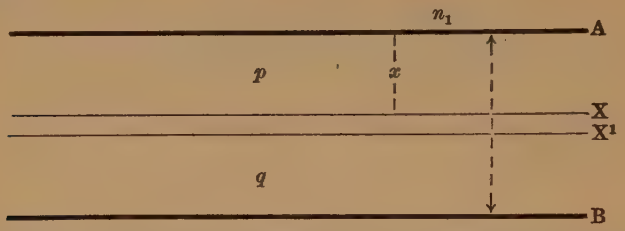
$$\int_0^l \alpha e^{\int_0^x (\beta-\alpha)dx} dx=1,$$

l being the distance between the electrodes.

II. Current between Parallel Plates

Let a number n_0 of negative ions start from negative electrode A.

Let p be the number of negative ions generated between A and plane X.



Let q be the number of positive ions generated between B and X¹. Consider two planes x and $x+dx$ from A. Pairs of ions generated between these planes are $(n_0+p)\alpha dx+q\beta dx$, where α, β are the coefficients of ionisation by collision for negative and positive ions respectively.

$$\therefore \quad \frac{dp}{dx}=(n_0+p)\alpha+q\beta.$$

If n=number of ions arriving at the negative electrode B, then $n=n_0+p+q$, or $q=n-n_0-\mathrm{P}$.

$$\therefore \quad \frac{dp}{dx}=(n_0+p)(\alpha-\beta)+\beta n,$$

$$\therefore \quad n_0+p=\mathrm{A}e^{(\alpha-\beta)x}-\frac{n\beta}{\alpha-\beta},$$

where A is a constant. When

$$x=0,\ p=0, \quad \therefore \quad \mathrm{A}=n_0+\frac{n\beta}{\alpha-\beta},$$

where $x=e$, *i.e.* at the positive electrode, $n=n_0+p$, for $q=0$.

Then

$$n=\left(n_0+\frac{n\beta}{\alpha-\beta}\right)e^{(\alpha-\beta)e}-\frac{n\beta}{\alpha-\beta},$$

i.e.

$$\frac{n}{n_0}=\frac{i}{i_0}=\frac{(\alpha-\beta)e^{(\alpha-\beta)e}}{\alpha-\beta e^{(\alpha-\beta)e}},$$

where i=current.

Pederson has given a more accurate calculation of a current obtained by ionisation by collision due to negative ions alone. It might be extended to positive ions also, but, owing to the complexity of the result in the present state of knowledge of these

phenomena, the accuracy is unnecessary. However, the calculation is instructive, and, later on, it may be found possible profitably to apply these more exact methods (see Pedersen, *Phil. Mag.*, **40,** 1920).

III. Townsend's Formula for α

The formula for sparking potential given in the text is obtained on the simplest theoretical assumptions and, as we should expect is not of much value, owing to the complexity of the effects. It depends on a formula,

$$\alpha = Ape^{-\frac{Bp}{x}},$$

where α=coefficient of ionisation by collision, p=pressure, x=field strength, A and B are constants. This formula is fairly satisfactory for electrons under weak fields, *e.g.*

x (field) . .	1000	900	800	700	600	500	400	300	200
α (experimental)	10·5	10·0	9·3	8·7	7·7	7·0	5·82	4·4	2·6
α (theoretical) .	10·1	9·7	9·3	8·6	7·7	7·0	5·55	4·3	2·34

but for higher fields, where the ions move faster, the assumption that they lose all their energy at collision no longer holds. Similarly, in weaker fields the velocity of agitation cannot be neglected in comparison with the velocity due to the field, and the formula no longer holds.

Schumann (*loc. cit.*) suggests that we should assume that

$$\alpha = ne^{-\frac{\beta}{x^2}} \quad \text{or} \quad \frac{n}{x^2}e^{-\frac{\beta}{x^2}},$$

where x=field strength, n=number of molecules per cubic centimetre. This gives a better agreement at high pressures, but its range of application is not very large and its theoretical basis is not very strong.

IV. Bouty's Theory

In an early experiment, J. J. Thomson showed that a gas at low pressure could be made conducting by induction from a high-tension oscillator. Bouty explains this particular phenomenon as due to a breakdown of the gas under a critical field strength, due to some process of ionisation not primarily one of collision. He made several experiments to determine this critical field strength, both by a direct method and also by introducing a glass bulb of gas between the plates of a condenser. When a sudden increase in capacity appeared at a certain E.M.F., the gas became

conducting, and the E.M.F. at which this occurred determined the critical field strength.

He found that this critical field (y) was related to the pressure (p) by the equation

$$y=ap+\mathrm{B}+\frac{c}{p^2}$$

(where a, B, and c are constants) at low pressures; and by the equation

$$y=a\sqrt{p(p+b)}$$

at higher pressures. The constant a was found to be characteristic of the gas, and in general, for chemically analogous gases, increased with the molecular weight. He called this quantity a the "dielectric cohesion."

Using thicker layers of gas and higher pressures, he found that under these conditions the critical field strength did not differ much from the ordinary disruptive field strength. Using the formula

$$y=a\sqrt{p(p+b)}$$

and approximating, we have

$$y=ap+\frac{ab}{2}.$$

If e is the thickness of the gas layer, $y=\frac{\mathrm{V}}{e}$, where V=applied potential.

$$\therefore\quad \mathrm{V}=a(pe)+\frac{a}{2}(be)=a\left[\left(p+\frac{b}{2}\right)e\right].$$

This formula obeys Paschen's law, except that the pressure is increased by $\frac{b}{2}$. Bouty thereupon divided the expression into two parts; one, which obeys Paschen's law, he attributed to dielectric cohesion, the other he called "dielectric adhesion."

In experimental results he found that the inactive gases had low cohesions, and that mixtures had a higher cohesion than the mean.

This theory is difficult to understand, for there does not seem any particular reason for it. The breakdown of molecules by an external field is not very probable, and the fact that the "critical fields" are the same as ordinary disruptive fields seems to suggest that the two effects are identical. What exactly dielectric cohesion means is not quite clear. The theory has been almost completely neglected in England and America, but on the Continent it is taken seriously by some.

V. Formulæ for the Field between two Spheres

The gradient at any point between two spheres is difficult to calculate and the result very inconvenient to handle. Methods for the solution of the problem will be found in Russell's *Alternating Currents*, chap. viii, and Russell, *Proc. Phys. Soc.* (December 1922).

The gradient at the surface of the spheres, *i.e.* the maximum gradient, is given by

$$R_0=\frac{V_1-V_2}{S}f_1+\frac{2V_2}{S}(f_1-f),$$

where

R_0=gradient at surface,
V_1, V_2=potentials of the spheres,

$$f=\sinh\ \alpha\ \cosh\frac{\alpha}{2}\sum_0^{\infty}\frac{\sinh(2n+1)\frac{\alpha}{2}}{\cosh^2(2n+1)\frac{\alpha}{2}},$$

$$f_1=2\sinh\alpha\cosh\frac{\alpha}{2}\sum_0^{\infty}\frac{\sinh(4n+1)\frac{\alpha}{2}}{\cosh^2(4n+1)\frac{\alpha}{2}},$$

$$\cosh\alpha=1+\frac{S}{2a},\quad S=\text{spacing},\quad a=\text{radius}.$$

Dean has given an approximate formula used in the text for the gradient at any point on the axis y from the surface :

$$R_y=\frac{V}{S}\left\{\frac{2S^2\left[S^2(f+1)+4\left(\frac{S}{2}-y\right)^2(f-1)\right]}{\left[S^2(f+1)-4\left(\frac{S}{2}-y\right)^2(f-1)\right]}\right\},$$

and f and f_1 have been calculated for values of $\frac{S}{a}$ from 0 to 2, and are given in a table in Ref. L/T16.

VI. Townsend's Theory of Current in Corona

Consider a wire of radius a inside a concentric cylinder of radius A. Let I be the current in the discharge per unit length of the wire, let q be the change per unit volume in the gas. We consider a positive corona, as the mobility k of the positive ion is proportional to the force, for large ranges.

If w is the potential at a point distant r from the wire axis,

$$I=2\pi rqk\frac{dw}{dr}.$$

Let V be the applied potential,

$$\frac{dw}{dr}=\frac{V}{r\log\frac{A}{a}}.$$

$$\therefore\quad I=\frac{2\pi qkv}{\log\frac{A}{a}}.$$

We now suppose that the space charge q is uniform, extending from $r=c$, where corona ends to $r=A$, and that the space charge in the actual corona is negligible in its effects.

The space charge q lowers the force at the wire by

$$\frac{\pi qA^2}{a\log\frac{A}{a}},$$

considering c to be identical with a for practical purposes.

Further, we suppose that this space charge reduces the force at the wire until it is just large enough to maintain the corona.

The force at the wire is therefore

$$\frac{V}{a\log\frac{A}{a}},$$

where V is the least potential necessary to give corona.

The force at the wire in the absence of the space charge would be

$$\frac{v}{a\log\frac{A}{a}}.$$

$$\therefore\quad \frac{V}{a\log\frac{A}{a}}=\frac{v}{a\log\frac{A}{a}}-\frac{\pi qA^2}{a\log\frac{A}{a}}.$$

$$\therefore\quad V=v-\frac{IA^2\log\frac{A}{a}}{2kv},$$

substituting for q in terms of I.

$$\therefore \quad I=v(v-V)\frac{2k}{A^2\log\frac{A}{a}}.$$

Schaffers * has obtained a formula more accurate for larger currents, namely,

$$\frac{v-V}{V}\log\frac{A}{a}=\sqrt{1+\theta}-1+\log\frac{2}{\sqrt{1+\theta}+1},$$

where

$$\theta=\frac{vIA^2}{kaX^2} \quad \text{and} \quad X=\frac{V}{a\log\frac{A}{a}}.$$

VII. Kunz's Theory of Pressure Effects

Consider a wire of radius a in a cylinder of radius A, and take the annular space enclosed between two cylinders r and $r+dr$ ($a<r<A$). If the pressure at r is p, p may depend both on r and the time t. The momentum gained in the time dt in this space is $neEdt$, if we consider that the ions which pass per second carrying the current give up their momenta to the gas molecules. E is the electric force at r, but

ne=the current=i,
e=ionic charge.

Therefore, further, this gain of momentum gives rise to an increase of pressure dp, where

$$2\pi rdp=neEdt=iEdt=iE\frac{dr}{v},$$

where v=mean velocity of ions.

Hence

$$\frac{i}{u}dr=2\pi rdp,$$

for $v=uE$ if u=mobility.

$$\therefore \quad \frac{i}{2\pi u}\log\frac{A}{a}=p_2-p_1,$$

where p_2=pressure at cylinder and p_1=pressure at wire. This effect has been observed by Fazel, and is known as the "ionic wind."

By the conservation of mass,

$$\int_a^A 2\pi rpdr=p_0\pi(A^2-a^2),$$

* Schaffers, *Phys. Zeit.*, **14**, 981 (1913); **15**, 405 (1914).

where p_0=initial pressure. But

$$p-p_1=\frac{i}{2\pi u}\log\frac{r}{a}.$$

$\therefore$ substituting,

$$\int_a^{\mathrm{A}}2\pi r\left(p_1-\frac{i}{2\pi u}\log\frac{r}{a}\right)dr=p_0(\mathrm{A}^2-a^2).$$

$$\therefore\quad p_1=p_0+\frac{i}{2\pi u}\log a-\frac{i}{4\pi u}\left[\frac{\mathrm{A}^2\log\mathrm{A}^2-a^2\log a^2}{\mathrm{A}^2-a^2}-1\right].$$

This gives $p_1=p_0$. Similarly

$$p_2-p_0=\frac{i}{2\pi u}\log\mathrm{A}-\frac{i}{4\pi u}\left[\frac{\mathrm{A}^2\log\mathrm{A}^2-a^2\log a^2}{\mathrm{A}^2-a^2}-1\right]=\frac{i}{4\pi u},$$

if A is large compared with a, p_2-p_0 is the pressure increase actually observed, and is proportional to the current.

Fraction of Heavy Ions

Suppose we have two sets of ions of numbers passing per second, n_1, n_2, and mobilities u_1, u_2,

$$\Delta p\text{ (the pressure increase)}=\frac{e}{4\pi}\left[\frac{n_1}{u_1}+\frac{n_2}{u_2}\right].$$

If we know u_1 and u_2 from mobility measurements, we may calculate the value of $\frac{n_1}{n_2}$, for $i=(n_1+n_2)e$, and the current (i) is known.

In this way the relative number of ions of different mobilities formed per second can be calculated.

VIII. Note on Initial Appearance of Corona

The opinion seems to be held in many quarters that it is Peek's view that actual corona, *i.e.* the visible glow, starts not at the electrode surface but at the "energy distance" from it, *i.e.* where the gradient has fallen to the critical breakdown value. This opinion appears to be based on Peek's hypothesis that "when ionic saturation is reached at any point a discharge starts"; since the ionic saturation is reached at the end of the energy distance, it is agreed that the discharge starts there. Peek, however, does not state this precisely, and the argument is hardly a sound one. If corona did start at this point it would hardly extend away from the electrode into fields below the critical value. Further, such a "dark space" has not been observed. In the case of a

conductor 5 cm. in radius it would be about 6·7 mm., and would be very noticeable indeed. The experiments of Whitehead, given in Peek's *Dielectric Phenomena*, are measurements of the visible glow.

Lastly, from the first the energy distance has been treated as a convenient hypothesis and not as an adequate theory. A real physical interpretation of the term "energy distance" is thus out of place.

Accordingly, it appears that the view that corona starts at a certain distance from the electrode has arisen from Peek's commentators rather than from the author himself.

In the text no such specific assumption is included. The treatment is independent of it, and it is believed that this omission better represents the ideas of the author to whom it is ascribed.

A good idea of the opinions current at the genesis of the "energy distance" theory may be found in the discussion at Cooperstown, 27th June 1913, given in *Trans. A.I.E.E.*, pt. ii (1913), pp. 1810 *et seq.*

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